

NOTE

“L’Addition, s’il Vous Plaît!” (No. 1)
“Bloch, Who?”

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ABSTRACT

Mathematical Defects in Solid State Physics are pointed out. The origin of the defects are traced back via experimental-trail up to the START line. The possible Features of the Reconstructed Theory of Solid State Physics are Projected on to the visual Map.

§1 Introduction

In 1932, Dr. Neumann published a book, the title of which was "MATHEMATISCHE GRUNDLAGEN DER QUANTENMECHANIK"; VERLAG VON JULIUS SPRINGER, BERLIN. They say he was a private lecturer, age of 29, when he published the book. Forty years have elapsed since he past away. But still, I found many things that I would like to mention.

We have four different kind of this book in Japan. Let me call them as, "The 1st kind, The 2nd kind, The 3rd kind, and The 4th kind of Dr. Neumann's Book". The 1st kind of the book is most frequently refered by Japanese Theoretical Physicists ; sometimes even by Applied Mathematicians. It is the translated book into Japanese, by the famous physicist Dr. Tomonaga. However, there are some troubles of this book.

First of all, it is "OUT OF PRINT". There is no glimps of re-publication at all. The reason of this quit is not clear, but one can easily imagine that the publisher can't expect to make a good business. So the trouble is, "New Commer" cannot find a copy on the market. What's more, The Old Theoretical Physicists often enjoy themselves by saying, "According to Dr. Neumann's Book and so on". They cite his work as if they were talking of the Holly-Bible.

The core of the TROUBLE is here. Surely, they must have their own copy in their home study. But as for a "New Commer", it is very hard to look at one. I noticed even "THE LIBRALY OF PHYSICS DEPARTMENT, UNIVERSITY OF TOKYO", doesn't have a copy at all! Obviously, "once upon a time", there must be a very diligent, young or old, theorist. He worked too hard until he forgot to return the book. (I may say he became a Tomonaga-Collection-Mania!) At any rate, chances were lost forever to read a book in The Library ; needless to say in a City Library.

The second trouble is, it is always true that "Even the most brilliant theorist, cannot be talented as a good translator, at the same time".

So, I had no choice but to pick up a copy of "Translated Book" into English ; what is known as "MATHEMATICAL FOUNDATIONS OF QUANTUM MECHANICS", translated by Dr. Robert T. BEYER. The book was published through "PRINCETON UNIVERSITY PRESS", in 1955.

To my great surprise, the book is unexpectedly "CLEAN!" It is hard to find "The trace of readers" beyond the Chapter 2. (This is because, they don't have a habit to distine the public book from the private one. They draw Under-Lines and/or jot sentences between the lines.) I, myself, graduated from this university in 1955. There should be about 1,000 students who graduated in these 40 years. So, I left with two possibilities to say ; there were only less than one percent students ever tried to open the book, or their reading-manner was changed so drastically into elegant way. However, soon after this encounter, I found PRINCETON UNIVERSITY PRESS issued the rivival Soft-Cover. So, I got one.

There is the "TRANSLATOR'S PREFACE" on the first page of the book, the date of which is "December, 1949. Providence, R. I.". My question

is following ;

1) Why do the TRANSLATOR of the book, Dr. Robert T. Beyer, so repeatedly mention as following? He says that “any deviations from the original text are also due to the author,” and “for his very considerable efforts in the rendering of the ideas of the original volume into a translation which would convey the same meanings.” Does this mean that there are big deviations between the English book and the German Textbook? If so, this will be a grievous trouble.

So now, I asked to a librarian to let me look the GERMAN copy ; which turned out to raise hell of the troubles to her. The GERMAN BOOK is so old. The date of publish is 1932, and it was moved into another place. Such a sort of place was named “Jail” by Librarians of University of Illinois, by the way. However, the jailer of Tokyo University were all peace-loving folks, and they were quite unfamiliar with the jailbird. Twenty minutes later, I found myself walking across the street to the old Physics Building, jingling a bundle of keys.

The cover of the GERMAN book was so dirtied with the human-hand-oil, However, I was surprised again when I opened it ; the pages were unexpectedly CLEAN after a few pages! Well then, when and where those Japanese Thoorists had learned that “Dr. Neumann layed the Fundamental Bases for Quantum Mechanics”?

I quickly made up my mind to forget the statements of those feathers. “The Jail” was full of musty air. By a quick looking, I didn’t find big “Deviations”. I found rather familiar but old fashioned formulae.

My next question is ;

2) Why Dr. Neumann didn’t give any “AUTHOR’S COMMENT” in the PRINCETON text? And, where was he in 1949, and what was he doing at

that period? The answer for the former part of this question 2) must be rather hard to make. It appears to be very likely that he was in Princeton. More precisely, he was a member of "The Institute for Advanced Study (IAS)"; the world famous institute for Physcs. By the word "A Member", I would strongly assumed that he was a full time staff of The Institute. However, as is the case for the most able scientist, he appeared to be very busy. He showed up almost every places in these Hell of the Days ! He got to go for a meeting at Atomic Energy Committee (AEC), since he was once a member of the Manhattan Project.

Besides, to my surprise and joy, he kept himself busy by the work using the COMPUTER for METEOROLOGY ! He was about age of 47, in 1950. Incidentally, I was age. of 29, and I was a graduate student and studying METEOROLOGY !

Before I'd talk about our (Japanese) case history of NUMERICAL WEATHER FORECAST, I'm very delighted to point out to the generous readers of this tiny article ; Dr. Neumann did two great things in his short life. He layed the sound GRUNDLAGE for the QUANTUM MECHANICS, by employing the HILBERT SPACE analysis.

AT THE SAME TIME, he employed himself as a FORERUNNER of the Modern METEOROLOGY ; which became the strongest champion that flew the banner of CHAOS. CHAOS became, in turn, the strongest oponent to the Newtonian Determinism. It is highly possible, that CHAOS will do its job again to the MODERN QUANTUM MECHANICS ! (Maybe, it will set up A POST MEDERN QUANTUM MECHANICAL THEORY)

It was only in 1986, when Dr, Lighthill raised a white hanky to the world. Since then, the author became very suspicious to, what they call, "All The

Established Theory". It is hard to anyone to get back to the innocent mind, once he swallowed the apple piece of suspicion. YEAH ! The author got a tendency to look into the Theories by grabbing a magnifier, or the scaling-factor. It turned out to be very powerful weapon to study "Infinity, Infinitesimals, Rational and Irrational Numbers".

I feel as if I am walking on a ridge of hi-mountain family, where the most fierce last battle between Inca and Chanka was held. The battle was achieved by throwing stones, and it had been over quite centururies ago. However, we are still rolling stones, named silicon-dioxied, aren't we ? Our battle of silicon is over ? Maybe. The author feels it would be better to clean up the old battle field, then.

In fact, the author still finds some ugly rocks and jolting monuments ; which might be better to kick'em out down to the bottom of the valley ! The author doesn't know whether he would be able to get it made. Some rocks've got even deep Roots like a plant ! He also noticed, that this job got to be done by myself, all alone. OK. No gains without pain !

§2 EXPERIMENTAL TRAIL

Before I talk with you on theoretical and mathematical side of this work, I'll let you know, what a curious thing was going around in the field of experimental side of the Solid State Physics.

As you know, about in 1912, Dr. Von Laue succeeded in to take the picture of "Laue Pattern" of Cuprous Sulfide crystal on the X-ray film. In order to Theoretical Physicists may understand what had been done, the simplest layout of "Laue Camera" is shown on Fig. 1 and Fig. 2. Fig. 1 is

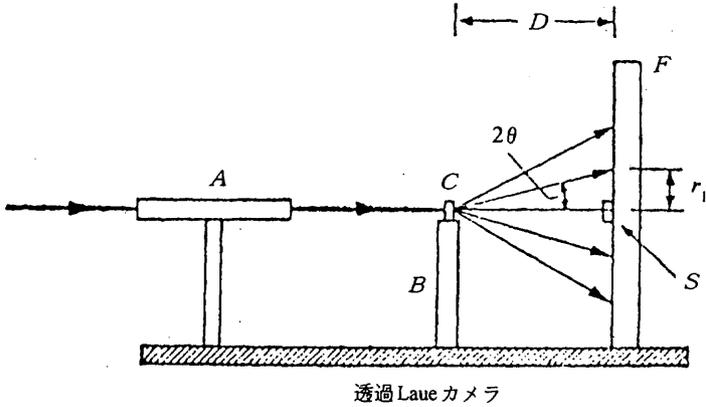


Fig. 1 The Layout of the "Transpiercing (Forward) Type Laue Camera".
(referred to Ref. 1)

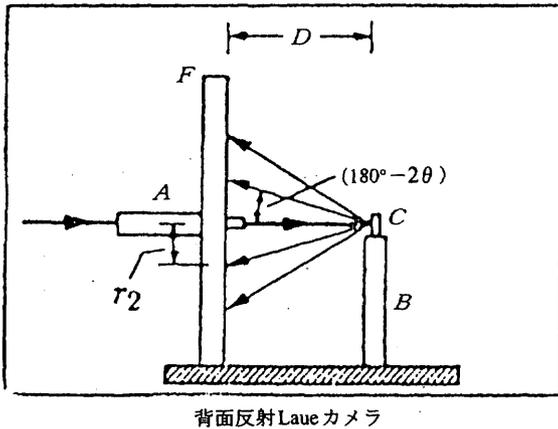


Fig. 2 The Layout of the "Back-Reflection (Backward) Type Laue Camera".
(referred to Ref. 1)

for The Transmission Measuremnt, and Fig.2 is for Reflection Type. [Ref. 1].

The "Laue Pattern" was soon analyzed by the contemporary physicists, Dr. W. H. Bragg and his son Dr. W. L. Bragg ; which turned out as the "Interference Pattern of the Diffraction of X-rays". If more mathematically speaking, the pattern is due to "The Interference of the Coherently Scat-

tered X-rays". (Right ?)

The point is that neither Dr. Laue nor Drs. Bragg never interested in the intensity of the scattered X-rays. Their main interests were, how to connect the pattern informations to the crystal structure. Later, as you know, many physicists focused their attention on the intensity distribution around the SPOT. The sophisticated X-ray Analysis were developed from this point.

In spring of 1966, when I was working for ISSP (Institute for Solid State Physics) as a research associate, I was asked to take pictures using the "Back Laue Camera" for TlCl. It was due to find the crystal axis, so that we could decide the crystal orientation for cutting a piece out of an ingot. At any rate, the point I would like to stress here is that, I was quite astonished when I developed a film, after, say couple of hours exposure to X-rays.

There were only two spots on the film, about 10×10 cm squared area ! So, I extended the exposure time up to almost six hours, and I got four spots, with a squeezed shape. Now I began to wonder about the lost X-ray beams. I was happen to get PhD by gamma-rays measurements, which was more or less similar to dosimetry of X-rays. [Ref. 2] So, I knew the X-rays intensity of 25 Kv, 20 mA, for more than 6 hours of exposure was not a joke. My question was, "Where all these rest of X-rays have gone ?"

I was figuring all the time through the boring exposure, sitting by the side of instrument, day after day. Finally, I found ; I drilled a hole on the center of the film ! The useless beams, or INCOHERENT X-RAYS, were happily escaped from the system ! In other words, "The Laue Camera" is The Detector which can catch up only COHERENT X-RAYS. (Could you agree ?)

Now, when Dr. de Broglie suggested the electron-wave, an experimental world-race began. They were anxious to take "The Diffraction Pattern" due to the Electron Wave. In Japan, somehow or other, Dr. Kikuti's so called "Kikuti Pattern" is preferred to rest of the world, and it is easy to find a picture in a Japanese text book of physics. Fig. 3 shows one of such examples. [Ref. 3]

Now you see, there are bright area in the central part of the Fig. 3. Nobody at that time intersted in the central exposure at all. Surprises were theirs ; "What a beautiful demonstration that the electron PARTICLE does have the WAVE properties at the same time!" Question is, "What is due to

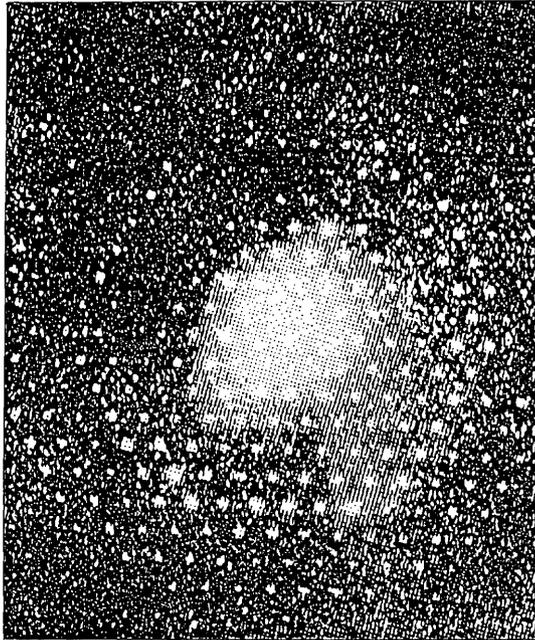


Fig. 3 KIKUTI's (Electron Diffraction) Pattern.
(referred to Ref. 3 ; which is actually, a textbook applied for the undergraduate students, general education, Tokyo University)

these PATTERN ?” Answer must be the same as for the x-rays’ case. It is “Due to the Interference of COHERENTLY DIFFRACTED electron wave!”

If you would look into an elementary text book for X-rays diffraction analysis, you would soon realize that the diffracted waves have the preferential direction to PROPAGATE within the crystal. The allowed direction is easily found by using “EWALD’S SPHERE” and the “BRAGG’S EQUATION.” This is connected, with the reciprocal lattice. We will see it again and again in BRILOUIN’S work, and BLOCH’S work.

Needless to say, any more, that the INCOHERENT WAVES can travel to any directions through crystals. This fact would be easily agreed if one would think about the penetration of X-rays, which has no big angular dependence to the crystal axis ; if not to say about Channeling Effect, or the Glancing Scattering.

I would like to point out here, that even the “Double Slit Experiments due to Atoms”, it is already shown that the Particle Waves (de Broglie Waves) were Partially Coherent. [Ref. 4, 5]

§3 THEORETICAL TRAIL

Funny Stories

As you may all know, Dr. Bloch derived the BLOCH FUNCTION as the wave equation that travels steadily through the crystal. [Ref. 6] He reached at the solution by solving “The 3-Dimensional Schrödinger Equation”, with the rectangularly Periodic Potential System.

He employed two more VERY IMPORTANT auxiliary conditions :

- 1) Separability of Variables.

2) Doubly periodic Boundary Condition.

(Double Translational Symmetry : He claimed this, since he wanted to apply The Group Theory)

[Ref. 6] [Von Felix Bloch "Über Die Quantenmechanik der Elektronen in Kristallgittern", ZS f Phys. Vol. 55, pp. 555-600, 1929]

However, to my opinion, he employed one more important Thought : He must had thought that "The Beautiful Pattern, (The Electron Diffraction Pattern), which he was looking by his own eyes, represented ALL THE ELECTRONIC WAVES IN SOLID". Another words, by the words of OPTICS, he didn't realize at all, that this is the REPRESENTATIVES of the COHERENT ELECTRON WAVES, *ONLY*. Well, I'm not blaming him at all. He was a theorist, and he should had not be able to imagine that sneaky experimentalists were cheating him by drawing a hole on the center of the film, or making shadows by the crystal itself.

As a matter of fact, when I was a graduate student and working with "The Liquid Ionization Chamber"(what a crazy thing in those days !), I noticed every day that The Electron Diffraction People were doing their experiments in the next door. They were using a tiny chunk of CdS as a detector for electrons, and they were doing very efficient experiments. But (!), I still remember clearly that they extended a slim rod, so that they can mask the central portion of the electron beam! All the INCOHERENT ELECTRON WAVES must had been stopped by that skinny rod! I don't know at all what was the trick with Dr. Kikuti's case.

At the end of Dr. Bloch's paper [Ref. 6], he says :

"Zum Schlusse möchte ich Herrn Professor Heisenberg meinen herzlichsten

Dank aussprechen für die Anregung zu dieser Arbeit sowie für sein stetes Interesse an inren Fortgang und die zahlreichen wertvollen Ratschläge, die er mir dabei hat zuteil werden lassen.”

A Hah! Professor Heisenberg was the famous MAGNETIC GUY! And the MAGNETICS are always interested in only ORDERED STATE (Coherent, in my terminology!). So, it's very plausible that they thought, “This is ALL!” (This is IT!)

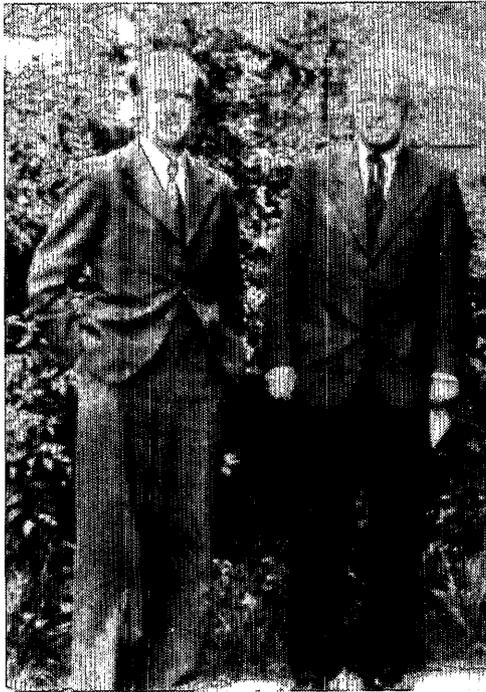


Fig. 4 Dr. Laue and Dr. Heisenberg (right to left).
(referred to PHYSICS TODAY, August 1995, Vol. 48, No. 8 ; “Bomb Apologetics”. The same pictures are shown in “Hitler’s Uranium Club”, AIP Press)

Ich Kann zu meiner Reisen
Nicht *mit diese Dunkelheit* :
Was soll ich länger weilen,
Dass man mich trib' hinaus ?
Lass irre Hunde heulen
Vor ihres Herren Haus! <Winterreise>

OK! Now I would point out "What they have done to the Solid State Physics" ? Firstly, why don't we talk about the "*Separability of Variables*" ? It is M. Jean Baptiste Joseph Fourier, they say, who invented to separate the variables to solve The Heat Conduction Equation ;

$$\Delta u - (\rho C/k) (\partial u / \partial t) = 0 \quad \dots \quad \text{[eq. 1]}$$

He invented this equation by himself, in order to apply for the mathematical contest held in 1811 by Académie Française de Science. He solved the problem by separating the variables and applying the newly invented Fourier Serie Expansion technique. Of course, he needed the proper boundary conditions, and I guess, he got it made by employing rather simple ones.

M. Laplace was quite good at, not only to solve the Laplacian Problem, he is said a sort of opportunist in political sense, and swam well across the whirlpool of French turmoils after the Revolution. However, he used to picked up the Separation of Variables and employing the Fourier Expansion Technique both for Solution Functions and Boundary Conditions. Everything is tedious and drowsy, but no serious problems were to be seen. (Good'ol days !)

However, when the END of 20th Century are drawing near, everything are

re-examined, sometimes by Computers, and people have noticed that we got to be more carefull to live in this world. Really, the last man of the Classical-Modern-Mathematics, the man of schizoid, M. Henri Poincarè, left many mines behind his steps, which began to detonate one after the other : Chaos and Topology! I'll show you how it's dangerous to separate varibales, without deep thought.

Dr. Bloch's TABLE MAGIC

I might recomend you to look into the text book, by Ashcroft & Mermin's, [Ref. 7], just for convenience's sake, rather than to look for the Bloch's original paper [Ref. 6]. There is no essential difference between Ref. 6 and Ref. 7 ; Ref. 7 is plainer than 6 and it has the air of up-to-date fassion, naturally.

There are three miserable things with Dr. Bloch's Table Magic. Due to these shabby instruments, no one would be happy to pay his/her fee any more, once he/she knew the TRICKs. I would open his 1st and 2nd tricks. But, I would rather leave the last one until we come to §5 HILBERT SPACE.

1) *Unstable Table*

Thanks to the Separation of the Variables, we've got plane-wave-like Bloch Function between the Lattice Points (real space). You think X-component and Y-component are independent. However, they can make interference. (Can't they ?) You would say, "We don't need to think about Phase for the Wave Function, since we would multiply the Complex Conjugate, later." OK. But, then, why do you happen to recall, all of the sudden, the Interference of the Wave Function after the Double Exit Slits Experiment ?

Because, they are the Coherent Waves, aren't they ? *Now look, your Bloch Function is so beautifully Coherent!* Their X-component, for example, stretches from $-\infty$ to $+\infty$ along Y-axis, with the mostly beautiful wave shape. Actually, this sort of interference were very famous for OPTICAL-HOLOGRAPHY people. I might introduce you just one of their work, “OPTICAL DATA PROCESSING” by Arnold Roy Shulman, 1969. [Ref. 8] On pages 690~692, Appendix 18, he shows us very interesting Figures, A 18. 1 (a)(b)(c) and Fig. 18. 2.

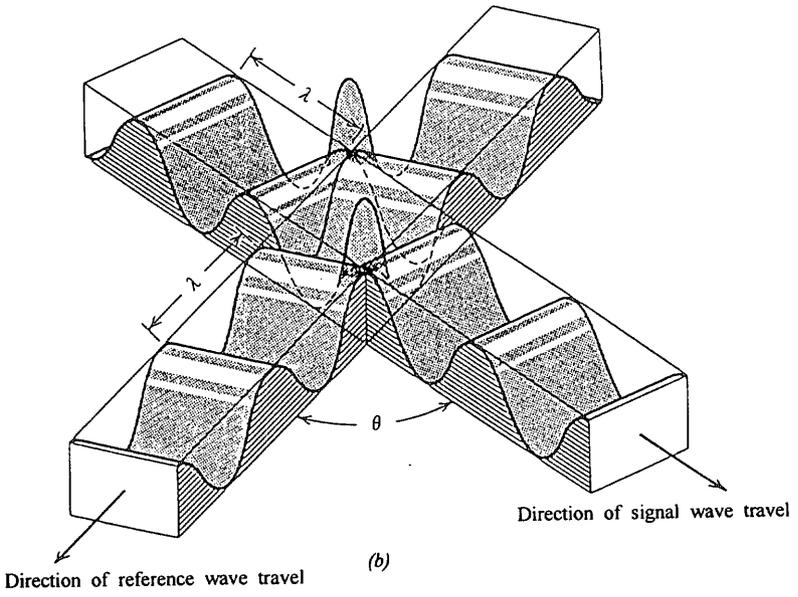
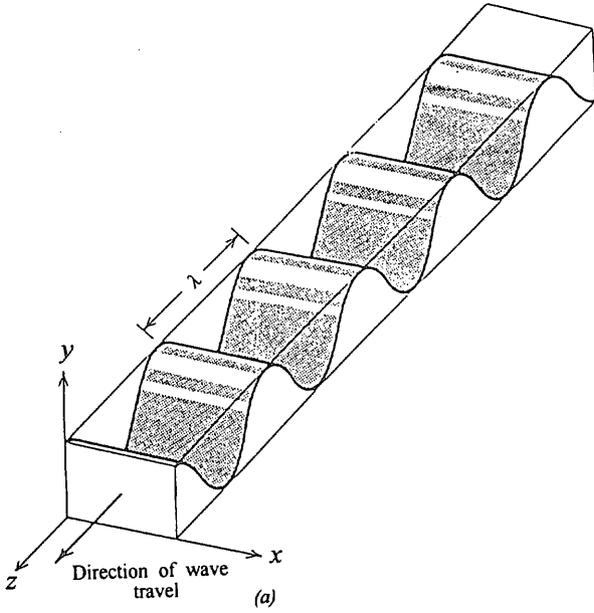
I might just borrow the Figures to show you, as Fig. 5.

As you see on Fig. 5 (c) you have a strong interference-peak at every corner of the crossing section. The peak travels down across the CRYSTAL, along the DIAGONAL direction. The peak gets on at every cycle of the BLOCH Wave. If I limit the Dr. Bloch's Table by down sizing to a single section as in Fig. 5 (b), then you see how his Table is Unstable ; at every cycle, the four legs at the corners make violent up-and-down motion! This is the different situations that theorists make sermons in his class : they say steady waves are running across the Table to X-Y-direction, independently [Fig. 6]. This is the result of Separation of Variables. You separated the variables, and you've got Coherence, which in turn generates the INTERFERENCE!

Later, we'll see the Separation makes the system mathematically INTEGRABLE. However, before we get to it, we'll see another shabby instrument on the Dr. Bloch's TABLE ! (Look, how they can be so simple minded!)

2) *Leaky Tablecloth*

Due to the well known “One Dimensional Periodocity”(for Bravais-Lattice), Bloch Equation is written down as follows [Ref. 7] :



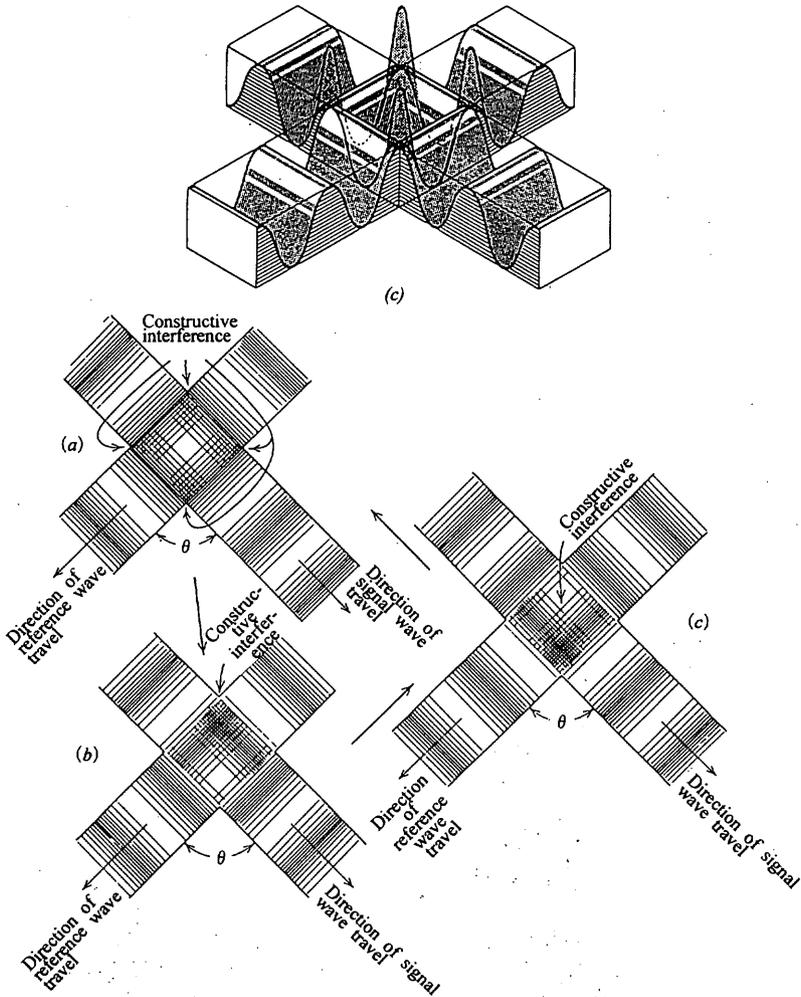


Fig. 5 Two Plane Waves and their Interferences.

(referred to Ref. 8) The point of argument is that the position of the "Intensity Maximum" travels horizontally to the diagonal direction. Due to the spatially fixed direction of these travelling maximum, the "Interference Pattern" appears (just as the same as the double slit experiments) when you put a detector screen at the position.

Another point to be stressed here is that if you would glare at a fixed position, then you would feel "faint away", because of the violent change of intensity from time to time.

$$\psi(\mathbf{r}+\mathbf{R}) = \exp(i\mathbf{k}\cdot\mathbf{R}) \cdot \psi(\mathbf{r}) \cdots \cdots [\text{eq. 2}]$$

where $\mathbf{k} = x_1\mathbf{b}_1 + x_2\mathbf{b}_2 + x_3\mathbf{b}_3, \cdots \cdots [\text{eq. 3}]$,

and $\mathbf{R} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3, \cdots \cdots [\text{eq. 4}]$.

And \mathbf{b}_i is the reciprocal lattice vector, satisfying

$$\mathbf{b}_i \cdot \mathbf{a}_j = 2\pi\delta_{ij}, \cdots \cdots [\text{eq. 5}],$$

where \mathbf{a}_j is the Primitive Vector for the Bravais Lattice. (Where indices 1.2.3 refer to the Bravais Lattice Primitive Vector ; for Simple Cubic Lattice, they end up with the crystal lattice vector, say, x, y, z, axes.)

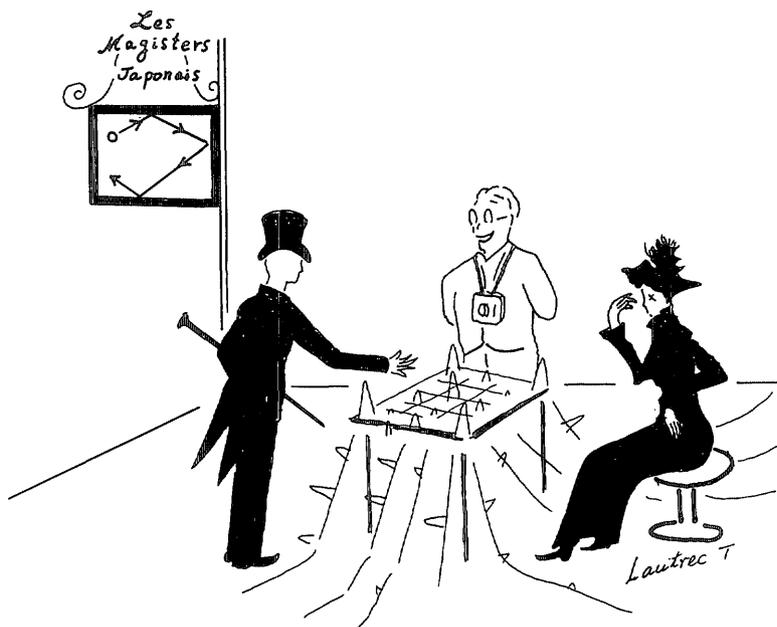


Fig. 6 Dr. Bloch's Table Magic Show, at "The Club LES MAGISTERS Japonais" (Referred to Lautrec)

The [eq. 2] is proved by Floquet in 1883, (Ann. de l'Ecole norm. sup. 2, X II, p. 47), and the "Differential Equations with Doubly-Periodic Coefficients" of ONE-DIMENSIONAL case, are shown in [Ref. 9] such as ;

$$f(z+2\omega_1)=f(z), f(z+2\omega_2)=f(z),$$

where $f(z)$ represents the circular functions $\sin z$, $\cos z$, $\tan z$, etc.

You might aware of it well that the Mathematics went on with the One Dimensional case, and it is your own risk to expand the results up to the Three Dimensional cases, with the help of Separation of Variables.

Thanks to the "Double Periodicity" (Notice ; No Solid State Physicist says in this way), they gang upon the REAL CRYSTAL. Their formidable justification is to employ "*The Born-von Karman's Boundary Condition*". Briefly speaking, this condition came from a theory of "THE ELECTRON GAS". Why don't we look into a TEXTBOOK for a while, and see what kind of theory US students are learning (and how Japanese PROFESSORS are chasing after them !).

In the "SOLID STATE PHYSICS" [Ref. 7], page 33, they mention as follows ;

"Thus, if our metal were one-dimensional, we would simply replace the line from 0 to L to which the electrons were confined, by a circle of circumference L. In three dimensions the geometrical embodiment of the boundary condition, in which the three pairs of opposite faces on the cube are joined, becomes topologically impossible to construct in three-dimensional space. NEVERTHELESS, the analytic form of the boundary condition is

EASILY generated. In one dimension the circular model of a metal results in the boundary condition $\phi(x+L) = \phi(x)$, and the generalization to a three-dimensional cube is EVIDENTLY

$$\begin{aligned}\phi(x, y, z+L) &= \phi(x, y, z), \\ \phi(x, y+L, z) &= \phi(x, y, z), \\ \phi(x+L, y, z) &= \phi(x, y, z).\end{aligned}\quad \text{eq. (2.5) in [Ref. 7]}$$

The author is sure to come back to this FALES statement, and should really enjoy the arguments in §4 SPACE and TOPOLOGY. But, before to do so, here is ONE MORE POINT that we can amuse. Why don't we begin with this?

Replacing L , the side length of the CUBIC crystal, by an integral multiple of a lattice constant a , (MORE rigorously, by $N_i \cdot a_i$), the TEXT says ;

$$\phi(r+N_i a_i) = \phi(r), \quad i=1, 2, 3, \quad \text{eq. (8.22) in [Ref. 7]}$$

After the game of double periodicity, they get,

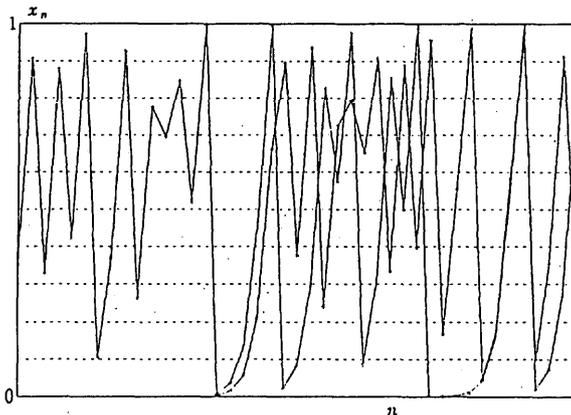
$$k = \sum_{i=1}^3 (m_i/N_i) b_i, \quad m_i \text{ integer.} \quad \text{eq. (8.27) in [Ref. 7]}$$

Now, the result of (8.27) is clear. All the numbers you may have is the Ratio of Integers (m and N), i.e., the result is known *Mathematically as "RATIONAL NUMBER"*! Of course, GOOD theorists aware of this, and they know it is dangerous to take ONLY RATIONAL NUMBERS into account. For example, in a Japanese textbook the careful author says, "since N is a sufficiently large integer, we'll have ALMOST continuous value of k , when m changes from 0 to N_i-1 ."

As you know well, the rational numbers were already discovered in Babylonian Age, by employing the 60-cycle system such as, $1/5 = 12/60$. Egyptians were good at factoring the numbers by the sum of fractional numbers such as, $2/5 = 1/3 + 1/15$, etc. [Ref. 10]

Could you be happy if you were regarded as one of the Egyptian Age Theorist? Sure you mentioned "ALMOST continuous". Let's check'em up "How Almost" it is, then. Let's take $L = 1$ cm, this is about the reasonable crystal size for an experimental physicist. And let's assume $a = 10 \text{ \AA}$ (1×10^{-7} cm); again this is a very common lattice parameter, I would imagine. Then you get the fraction $\Delta(m/N) = 1 \times 10^{-7}$. So, you've thought your Mesh Table Cloth is FINE ENOUGH !.

Alas! Good'ol days were gone forever! And the Poincaré's mines did detonate : CHAOS BREAK IN! As an simplest example, I'll show you on Fig. 7, [Ref. 11], how it is a sensitive and awful thing! The only difference of



二つのきわめて接近した初期値 $x_0 = 0.35$,
 $x_0 = 0.350001$ による数列

Fig. 7 An example of Chaos due to the Logistic Projection.
 (referred to Ref. 11)

0.000001 over 0.35 for the initial condition, makes the desperate discrepancies ONLY after 15 steps of motion. You may say the ratio $0.000001/0.35 = 2.86 \times 10^{-6}$, so your mesh is still finer than this. However, this is just an example for students. And the real chaos takes place with much smaller difference after forgetably long elapse of time !

Still, you may whistle in the dark "What's on earth my Electron Theory on Solid State Physics to do with Chaos?" Precisely ! Here came in the AKP (Anisotropic Kepler Problem), these days, and soon peoles will discard your mesh unless you fix it before they would abandon it. More acurately speaking, they would try to find the reasons why trajectories were so deviated each other, nevertheless they started from Infinitely Close Points in Phase Space. Poepole will be easily disappointed if they cannot get closer than 1×10^{-7} . They may utter, "We cannot play THE GAME OF CHAOS at all, on such an old-fashioned Spring Mattress !"

Another theorist may say, "It is not $1/a$ but $h/a!$ Look into Dr. Shockley's famous book, for example! You see the factor h is awfully small ; Plank's constant $= 4.1357 \times 10^{-27}$ erg \cdot sec."

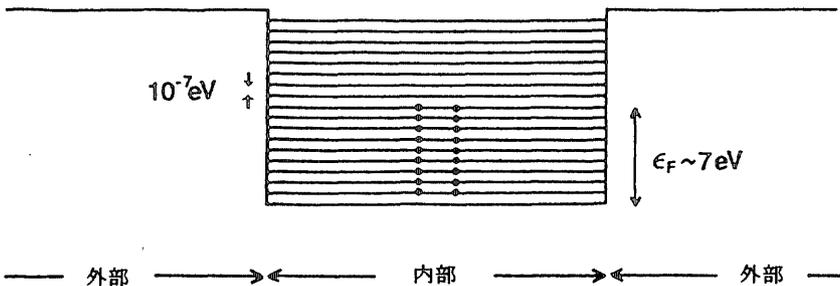


Fig. 8 Energy separation of electrons in metal.
(referred to a open-lecture note, performed in Japan)

If you thought this statement has any meaning, you are NOT A THEOREICIAN, but A THEORIST! If you were working for a University, you’d better close up your notebook and get out of your office, leaving the office key on the desk ! Ever since, M. Cantor thought about the continuity of functions, we said farewell to Egyptians! It happened on 1870, and it is only you that didn’t realize it until now ; what an idle guy is dwelling in Webs World Widely !

I might give you a homework, then, to save your face ; try to calculate a scattering problem of your favorite electron on your Mesh-System, under the k-vector conservation rule. You would easily agree that you need REAL NUMBER to calculate “The Length of a Vector”, or “The Norm in Euclidean Space” !

For other sort of theorists, they appear to be interested in only FERMI SURFACE related business. For example, a world famous Japanese theorist mentioned in his open lecture as following ; “For a metal, the excitation energy for electron just under the Fermi-Energy is order of 10^{-7} eV, and we can regard this as a matter of fact ZERO”.(cf. [Fig. 8]) Alright, he’ll never know 21 Century !

§4 SPACE and TOPOLOGY

As we have read, the textbook [Ref. 7] said, “the transport of charge and energy by the electrons is far more conveniently discussed in terms of running waves. If our metal were one-dimensional, we would simply replace the line from 0 to L which the electrons were confined, by a circle of circumference L.” The authors of this textbook use a word “topologically” in the same section. However, it appears to me that they are not at all good at “TOPOL-

OGY”.

The POINT is , they made a RING out of a STRING, and THEREBY they raised the DIMENSION of argument from ONE to TWO-DIMENSIONAL-SPACE. (You see the POINT ?) OK! Then, it is the most elemental knowledge of Topology, that “In order to have steady flow on the Two-Dimensional-World, there must be at least One-Hole on the Surface.” This is one of the most fruitfull application of Topology. So, I don't know who was the first that made a metal-ring out of the string-metal. But, he was the most luckiest or inocent person who didn't stepped on to the Poincarè's mine. You would be easily agree with the rest of the world, if you just pay a look at Fig. 9.

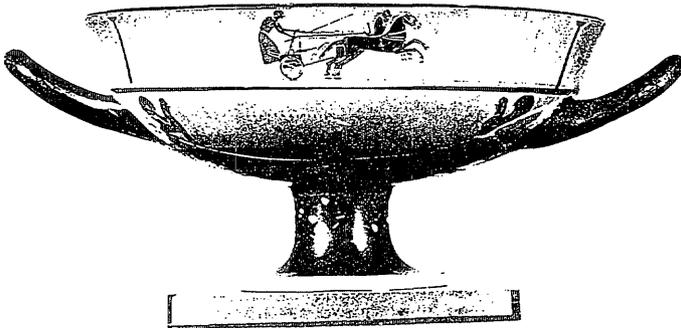


Fig. 9 The Strongest Evidence for Steady Flow in 2-Dimensional World.
(The wheel can rotate steadily, because there's a hole at the center ; which accords perfectly with the Topological Demand)

This fact is, surprizingly enough, firmly stated by a Saint RAO-ZU (老子), about BC 800 ~ BC 500(nobody really knows, when and who He Was). He left only about 5,000 chinese characters in his life (!) In his Chapter 11, he says, “三十幅共一轂。当無用，有車之用。(30本のスポークを集め，車輪を造る。真ん中が空いているので，車輪が廻せる) Putting 30 spokes together, people make a wheel. There is a HOLE at the center, THEREBY it can be

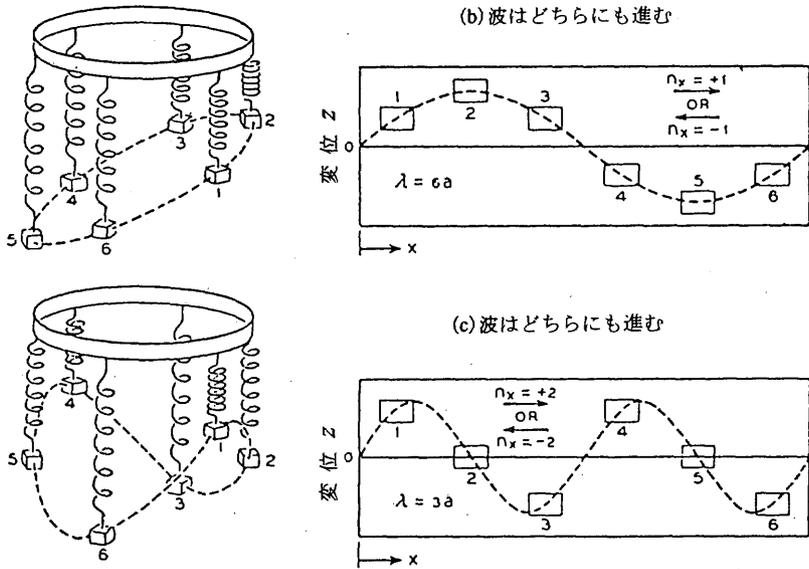


Fig. 10 Shockley's Circulating Current.

rotated!". "Monsieur Poincaré, écoutez bien?"

The second Lucky-boy is, of course, Dr. William Shockley. In his monumental book [Ref. 12], he says, repeatedly, about a running wave by showing Figures such as, like [Fig. 10]. What's more, he mentioned, "However, our main interest in quantum-mechanical theory centers about the problems of electrical conductivity rather than those of cohesion, and etc." Conduction is One-dimensional problem for the Experimentalist's point of view. Shockley was real ingenious, since he luckily circumvented the Boltzman Equation. I should be back to the modern conduction theory of which people are arguing that "Conduction is Transmission".

NEXT POINT is this ; In Three-Dimensional-Space the argument above won't work so EASILY. For example, we saw in the textbook, they (and Dr. Shockley also) said "EVIDENTLY, it is easy to generalize the Bloch Func-

tion and the periodic boundary conditions up to Three-Dimensional Space.” Obviously, there is no problem at all, mathematically, to invent the Three-Dimensional Periodic Conditions. You separated the variables (x, y, z, AND t), and solved the problem so nicely. Now, you got to combine the results! This is the HARVEST of the theory. Then you see, you stepped onto the MINE !

I noticed you've separated so easily the space coordinate, x, y, and z. But, it turned out OK. And of course, you've DROPPED time-coordinate t, since you saw it is so easy to SEPARATE [t]. But, it appears that you never realize the importance of the [t-axis] : it is related with MOTION, or MOVEMENT, either in real space or in PHASE SPACE. [Time and Space] have completely different meaning in Non-Relativistic World. Of course you know (savoir) it well, but you didn't understand (comprendre) it well.

It is at this point that some theorist appears not to have keen sense of SPACE : We have to face the four Spaces ; Real Space, Configurational Space, Phase Space, and Mathematical Space. Let me ask, “What on earth the K-Space is ?” (I'll be back at this point later). At any rate, no sooner than you put the t-axis back, then you've got the running MOTION! Remember, everything got started to MOVE! (on the magic table).

Now, TOPOLOGY says, “For Three-Dimensional Space, there is No Steady Flow, Except the space had *The TORUS shape!*” This is also very elemental knowledge of Topology. You would be able to convince yourself, if you look at [Fig. 11]. This is the most fruitful and practically effective conclusion that such a useless mathematics as topology could have reached! So long as you would insist to live within the Three-dimensional EUCLIDEAN space, you have to perceive that the world is Topologically Equal to the SPHERICAL world. There is no steady flow on (and within) the sphere.

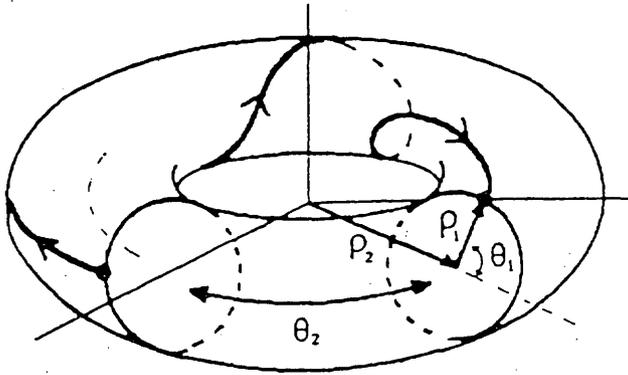


Fig. 11 The TORUS World, which people have to live in ; if he wants to have a steady circulation at the same time.

BECAUSE this is the most formidable "Brower's FIXED POINT THEOREM". You may found easily that the Wheel Mr. RAO-ZU argued was actually three-dimensional, and you can drill a hole along the axis, which drill through the Two-Fixed-Point on the surface.

I'll show you how people staggered at this point, simply because they did

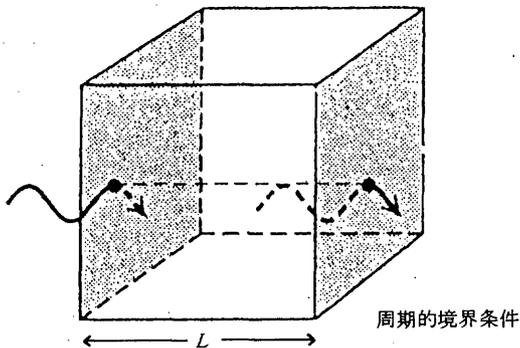
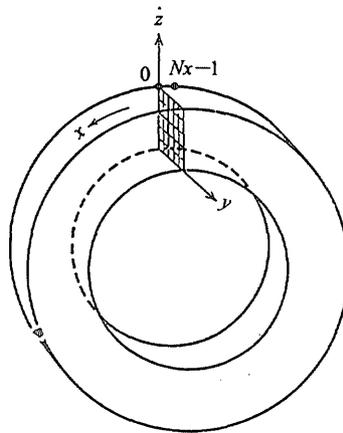


Fig. 12 An example of CUBISME ; who tried to solve the Tri-Lemma without keeping his weather eye open for Post-Modern-Science.



結晶を x 方向にまるめてつくった環

Fig. 13 An example of SURREALISME ; the author recommend to readers that they might ask M. Dali, instead of to Dr. Einstein, to find their own TIME.

not keep their weather eye open for Modern (French-American) Mathematics. Figure 12 is the most innocent example. But the author was clever and careful enough that he did not show except one-dimensional flow [Fig. 12].

For another example, I'll show you the most fancy Figure on [Fig. 13]. I don't know whether this is the result that he invented by himself, or he was taught by some BAD THEORISTS. At any rate, he showed us that we got to live in the TORUS WORLD (Great !). True ! Maybe we are in a torus space ! But, it is about more than thousand million light-year size, that cosmo-physicists are dreaming ! (sigh !) So, it is just a "Laughable Joke" that a crystal-sample we made has a Torus character !

Incidentally, [Fig. 14] shows Dr. Shockley's figure of k -vs-real space, what we call Brillouin Zone. Shockley was not so brave enough, obviously, to polish up his crystal into Torus shape !

Actually, in the Torus Crystal in Fig. 13, even the primitive "Translational Symmetry" is broken. I might ask you to strengthen your imagination just

Brillouin 領域と量子数としての結晶運動量 P

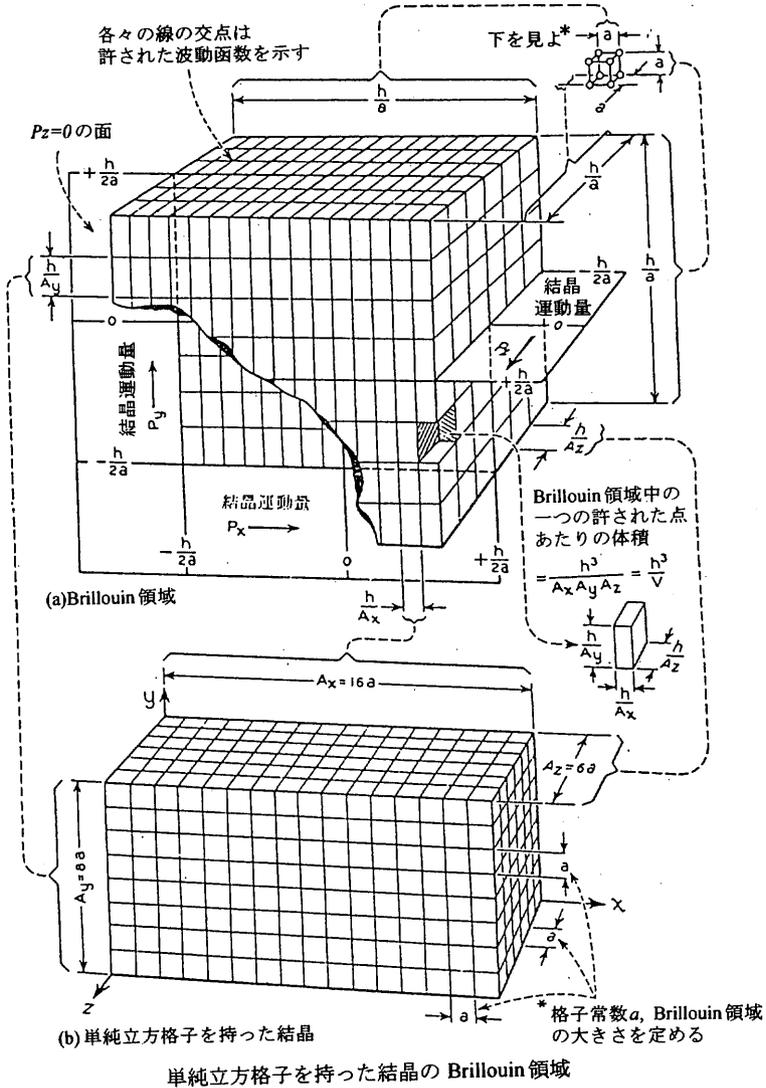


Fig. 14 Dr. Shockley's Brillouin Lattice. (referred to Ref. 12 : All of the sudden, he declared, "Our interest is conductivity". Ingenious! He must had felt Ambush !)

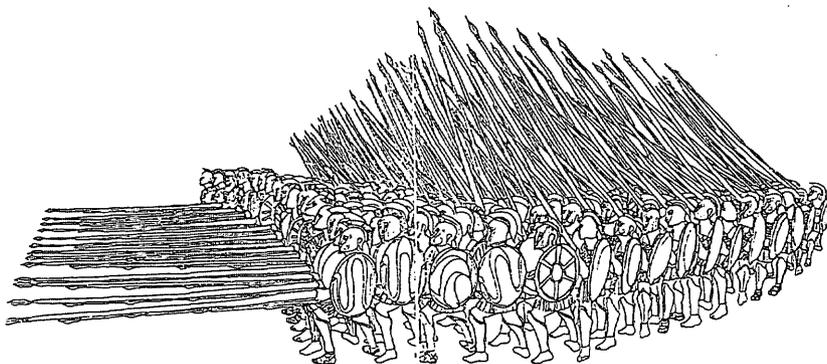


Fig. 15 An example of Translational-Symmetry System.
(There are $16 \times 16 = 256$ (!) Square-Infantry)

by looking [Fig. 15]. It was the very common knowledge even for the Sqaure-Infantry of Persian Army in Greek Age ($16 \times 16 = 256$!) ; “How difficult it is to turn, even gradually !”. This is one of the important reasons that Roman troops conquered Mediterranean ! He should had been better to watch the famous mosaic, named “Battle of Issus” or to pay a visit to Paris, on 14 Juliet, and watch how beautifully the cavalry manages on Champs-Elysee or at Rond Point !

When I was working with Prof. Frederick Brown, I happen to watch a movie named “Feynman Lecture”. He was at the point to explain the running free electron in a lattice structure. He walked quickly to the left end of the platform, hid himself behind the curtain, then suddenly re-appeared from behind the curtain on the right end with funny face ! (this was a movie). All the students in the class, more than 50, burst a laughter ! However, when I tried to find this Torus problem in his book “Fynman Lecture” later [Ref. 12], I could not find even a word for this magic! What a clever guy Prof. Fynman was ! He circumvented the Poincarè’s Mine, and all the student who watched the Movie was goofed.

§5 HILBERT SPACE

It was about in 1980, I found an article, written in Japanese, on Theoretical Electronic Problem in an academic journal. A Famous Professor in ISSP said ; “I have gotten The Almighty Key. (He meant Quantum Mechanics) There were NO DOOR that I could not open by the key !” I can still clearly recall when I was almost burst into a laughter in the darkness of the Lab., alone! (I was working on the optical measurements, such as reflectivity measurements, and I always kept the room dark to avoid any stray light).

“What a wooden-headed professor he is! Can he be one of [The Best and The Brightest] of Japanese Intellectual Society ?” It was just a laugh at that time, but now, it turned into anger and/or sadness! What a deep dent he left in our country !

You know, nothing can be concrete forever or can be Almighty ! Needless to say about Physics. Physics is much more Fragile than mathematics ! And now, look, even the Newtonian Mechanics were laid down [Ref. 14], how can it be safe even with the Almighty Key ? The tighter you stick to the Correspondence Principle, the sooner you would find the *Curiosités misérable* !

In order to make long story short, I would confess my current understanding on Mechanics and/or Dynamics (may it be Classical or Quantum). There are 4 SPACES that we must concern :

- [1] Configuration Space.
- [2] Phase Space.
- [3] Mathematical Space.
- [4] k-Space.

【1】 *The Configuration Space* is composed by ONLY q-axes. It is called rather frequently as “The Real Space” by Japanese Solid State Physicists ; (Hereafter, author writes in a short form JSSP). This is because, for JSSP, arguments are very often related with the “Real Axes”(X) and Time (t). Time (t) is, however, hidden as the “Implicit Parameter”.

The Configuration Space has the simplest meaning from the Mathematical point of view :

Case A) It is the “Number-Vector-Space”, which is composed by the “Real-Number-Axes”. The dimension of the space can be increased from 1 up to n (n can be infinity, if you need it.) Here, however, n is the Integer. (You cannot set up $n=\sqrt{2}$ dimension space). Therefore, n is the discrete number.

It is in this space that the Classical Mechanics flourishes over us. Many nice Shows were taking action in this Theater, which were Physically beautiful but logically questionable ; such as “small action causes small deviation”. This is simply the result of linear algebra (definition of vector), and linear operator (linear differential equation or the linear Number-to-Number projection operator).

If you reduce the Number of Particle of this Play down to one, then you have common Euclidean Space (3-dimensional), and the Single-Particle-System. JSSP fond of this system, somehow or other, and they call this as “The Real Space”. Students were easily cheated by the word because the system is really Real Space (3-deimensional), and forget about they are learning only about Single-Electron-Approximation.

Case B) It is possible, and mathematically nothing strange, to crank up to build “The Function-Vector-Space” within the same category. This is easily

understood, if the Functions saticefy the same rule which was imposed upon the Number-Vector : Linear Algebra and Linear Operater.

However, it is impossible now to represent the Function-Vector by a pointing arrow. It is, therefore, understood by the words of "SET", "PROJECTION", and "MAP". Of course, the Base is in the Real-Number-Set.

If we recall the Case A), for Single-Particle-System, the motion of the Particle is introduced by changing the implicit prameter (t), thereby changes the cordinate, or the position on the real line. At the same time, there can be the case such as the Base of the Euclidean Space itself do rotates, or displace depending on time (t). Then we have the relative motion. Especially, when we put the origin of the relative system upon the particle itself, we get to the noninertial system ; Meteorology is one of the example for such a case.

Mathematically speaking, all the case mentioned above is simply summarized as "A Number-to-Number projection by an Opetator (functions)". The same idea can be used for the Function-Vector-Space. In this case, however, we have two types of projections ; Function-to-Function projection and Function-to-Number projection.

In Quantum Mechanics, The Schrödinger's Wave Function is ASSUMED to saticefy the Function-Vector conditions ; such as the "Superposition Principle". All the gravely told Physical Essence by JSSP is nothing but they declare that "We have decided to dwell in the world of Linear Algebra !" What's more, sometimes we hear terrible (laughable) confusions with Linear Algebra and Linear Operetor ! They lack the sense of SET, PROJECTION, and MAP. At any rate, my intent here is to point out that Quantum Mechanics is contained into a very limited small area, if you look it under the spot light of mathematics. I might come back this point later

again.

[2] *The Phase Space* is the most popular Space, for the Mechanics. It must be composed, DEFINITELY, with p and q , (if any). JSSP likes to draw a phase space when the system is an easy one, such as the Hamiltonian can be described by a Simple Harmonic Oscillator. However, they suddenly turn their backs when the system becomes a hard (complicated) one. They scarecely talk about the Poincarè cross-section. This is because their knowledges came from the English-German-Physics school ; they scarecely learn French. They didn't even realized that Dr. Einstein spoke French when he went to Paris, and/or he talked with Poincarè by speaking French. However, this attitude was not limited to JSSP. Since the Main People of the days, who got together around Copenhagen, decided to not to think about (Gave Up) the MOTION of the system in either Phase Space or in the Configuration Space. There was a hidden reason, I suspect, that they decided to toss away to think about Motion and/or Trajectory of a particle. I suppose, this is because Dr. Heisenberg DISCOVERED the "Uncertainty Principle" before every-thing : Which is nothing but A Mathematical Feature of Space and Operators.

Once the phase factor shows up to make sense, all the Castle of Cards, such as Uncertainty Principle, will fall down at the spot. This is very likely to happen in the field, where Hamiltonian depends on time- t , or at the field where irreversible process may become the main interest ; Chemical reactions, Chaos could be the top running candidate.

[3] *The Hilbert Space* is nothing to do with, directly, to the above stated Spaces ; if you would allow me to speak in an OVER SIMPLIFIED

way. However, I would state some elemental or well accepted mathematical results, in order to show you that I am not a simple idiot nor an inhabitant of a bughouse.

The Hilbert Space is a rather simple space, or more elegantly speak, a beautiful space than the above stated spaces. This is because, the Hilbert Space is a Mathematical Space, and the rest of the spaces are more or less Physics-Related dirty Space! Look! The Goddess of Mathematics is standing in the center of the group of Goddess (meeting ?), and the Physics Goddess is a little bit put aside, and she is "less-beautiful". So happened on the Minkofsky-Space to the Special Relativity. I learned Dr. Einstein was once a student of Professor Minkofsky, but I don't think Dr. Einstein ever tried to look for some example to dedicate his work to Professor Minkofsky. On the contrary, I've read Professor Minkofsky said that "His mathematics (geometry) is terrible, and I have to help him to fix it up!" I don't stick at all whether this anecdote were true or not. What I am trying to say is that "Mathematics is mathematics, and Physics is physics !" At any rate, the nature was full of complexity, and the beautiful Minkofsky Space was left behind, when Dr. Einstein stepped into the general relativity. So did happen on Galileo. He was right when he murmured "Still the earth moves!" However, very at this point, Galileo was wrong ! The Mechanics that he insisted cannot be true on the Rotating System! (Non-inertial system).

The same sort of thing could be happen with the highest possibility. We are almost facing to it. So, "all physicist that take the mathematics shall perish with the mathematics." (Because not only the strong points of argument, but also the weak points are revealed, to the broad daylight by mathematics).

The Hilbert Space is a Set of Function-Vector and Real Number, very shortly speaking. However, the Function here means not such a simple function as before. It is defined by a set of Inner-Product of Function-Vectors ; by inner-product, it means the Integral-inner-product ; by integral it means Lebesque Integral, so that you can D-Tour the Discontinuity ; by discontinuity it means that you must take care of the Unbound functions, but they must be Complete etc. But still, it is old in the sense, that it lacks the Side-Winding Topology ! (All squadrons ! Take off ! Go! [Ref. 15~Ref. 25])

What's more, you would be surprized if you can look through the Nebel (fog), that how strictly Dr. Neumann squeezed his territory narrower and narrower. For example, the inner-product is limited only to Conjugate or even down to self-conjugate, (adjoint). At any rate, Dr. Neumann is said that he was also a student of Professor Hilbert, and I would suspicious he would be a bit in a hurry to write a report to Professor Hilbert.

What's more-more, this is just a story within the Hilbert Space, and there is another story to project the SET to the Map (as I told you before). Maybe you have understood well. The Hilbert Space is the Linear-Algebra-Space. However, the way of projection is nothing to do with the Algebra. This is the matter of Projection Operater, or the matter of Physical Equation. If you want to say more realistic, this is the Space where Physics and/or the Real World (Nature) CUTs-IN !

What's more-more-more, They (who ?) assumed that the nature of the projection is LINEAR ! As I told you before, even at this very elemental JCT, there are incredible mis-handling professors who lead students into the WRONG WAY. There are century long arguments, whether the Perturba-

tion Theory can patch up the discrepancy between Nature and (Non-linear) Mathematics.

As a small conclusion, there is not a bit of guarantee that the Function-Vector in the Set of Hilbert-Space can be composed by the Schrödinger's Wave Function. Then what does it mean Dr. Neumann wrote a book, "Mathematische Grundlagen der Quantenmechanik" ? You might ask Dr. Neumann directly, "Why didn't you put your Name and Preface Words onto the translational book, published at the Princeton?" This question may let him to BUTTON his lip.

I don't care what JSSP may say on the world of tiny-size, such as atoms or molecules or much much small particles. However, it appears that they made a fatal mistake to open their front to the other side of the world. Another words, to the large-scale world such as Solid State Crystal. I must say, (or better say) I must confess, that I am no theorist ; therefore I really don't know how far Quantum Mechanics can go down to small world such as Quark or less. But when it comes to Crystal, or Optics, that is the prairie where I was used to run. I won't let you allow any more, the fly by night theorists, to vend Antiques in this Greenfield.

You made the fatal mistake, Doctor Fell,

The reason why I cannot tell ;

But this I know, and know full well.

I do not like thee, Doctor Fell. ⟨Mother Goose ; Doctor Fell⟩

Who told you, Dr. Fell, that "The World is composed by molecules, and molecules are composed by atoms. THEREFORE (!) QM governs the whole

WORLD ?”

[4] *k*-Space is the most mysterious space of the four. I really don't know what it is. Dr. Shockley says in his famous book, “Electrons and Holes in Semi-conductors” that this is the Crystal-Momentum Space. I tried to dig up my old lecture-note when I attended Professor Muto's class on Solid State Physics. He made so many REMARKS in every lecture, so the note book showed the arrays of his remarks. On the final line of the lecture, which was performed on May 11, 1960, in Komaba Campus, University of Tokyo, Professor Muto said ;

“Let's think a little bit about current. It is defined by,

$$-e\langle v \rangle = \langle j \rangle \quad \therefore \langle v \rangle = (1/h) \text{grad}_k E_n(k) \quad \text{and} \quad \langle v_x \rangle = (1/h) \partial/\partial k_1 E(k).$$

This somehow looks like Canonical Equation,
such as $\partial q/\partial t = \partial H/\partial p$.

Therefore let's call hk as “quasi-momentum”.

The reason why I call it “quasi” is, if we operate- $i\hbar$ grad, ie p_x ,
then ;

$$p_x \psi_{nk} = (\hbar/i) \partial/\partial x [\exp(ikr) \cdot u_x(r)] \neq a\psi ;$$

which is not equal to ψ as you see it.

Therefore, this is not the EIGEN VALUE! (function?)

Were it happened today, I would never let him go out from the class room, without putting upon him the shower of questions. But, all I can recall is the rather tired and drowsy ending time of a lecture. And I don't think Dr. Imai, who had suggested me to attend the lecture, made no questions, either.

Later, Dr. Imai also invited me to attend with him to Professor Kawamura's lecture on Semiconductor Physics, performed at Hongo Campus, University of Tokyo, and that made me began to think about more seriously on Solid State Physics.

There are so many POINT DEFECTS in the Theory of k -Space. It make me DARNED, even if I just to think about it :

- 1) It is just a k - ω space, which can exhibit the dispersion of the wave vs energy. There is no guarantee at all that de Broglie-wave has the same dispersion as light. On the contrary, de Broglie himself realized rather late time that Material-Wave is different from Electromagnetic Wave.

However, when I tried to show people about its dispersion, by analysing an Atomic-Double-Slit-Experiment, with the possibility of its Partial-Coherence, all JSSP, including who have done the experiments by her/him self, neglected the arguments. The only person who reacted more or less active was Dr. Michel Grosman in Strasbourg, France.

- 2) Mathematically, the k -axes are composed by RATIONAL NUMBER ONLY, which carry us back to the Egyptian Age. The results comes from the wrong Boundary-Conditions, as I've shown.
- 3) There is no reason that they can be safe to assume that the functions must be Bound, Single-Valued, Regular, (continuous) etc. When it comes the dis-continuous functions take places into the play, then even the Fourier Expansion cannot be guranteed, after Cantor discovered the break down.
- 4) Once the phase factor came into the system with meaning, then all the stories build upon the Castle of Cards will break down. The first one

is the Uncertainty, of course.

§6 Temporal Conclusions

- 1) The Born-von-Karman's Boundary Condition is False.
- 2) The running steady wave, such as Bloch Wave, cannot exist within the 3-Dimensional Euclidean Space.
- 3) Bloch's Wave Function is derived by forgetting the Incoherent Component of Wave. Therefore, it is just a fiction.
- 4) Separation of Variables ended up with to get infinitely Coherent Length ; both in Space and Time. It sounds like a Fairy Tales ; where the space left for us to live ?

Due to the above stated Four Grievous Faults, Bloch's Wave Function got lethal Dents.

- 5) Dr. Neumann's Hilbert Space Theory should be re-examined ; especially, his domain of Set, Relations to Non-linear Projection Operators, including Chaos, must be cleared up (Alles ist in der Nebel !). It got to be made crystal clear that through which rout The Logistic Map, that (may) have led us to the Ir-reversability [Ref. 25], could find the way to his Set.

Finally, the author might show a visual Rout-Map, for readers' convenience's sake [Fig. 16].

MATHEMATICAL SET

PROJECTION (PHYSICS)

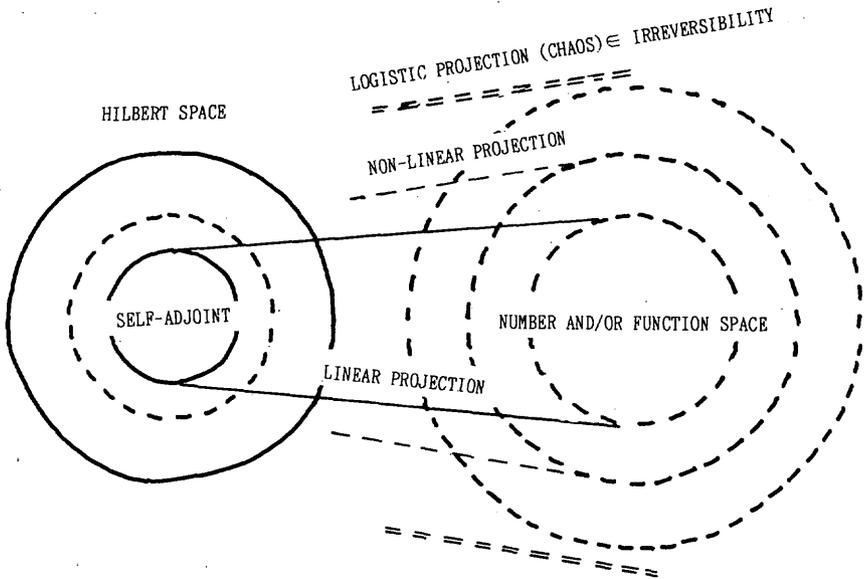


Fig. 16 An example of Possible-Rout-Map, for the Post-Modern Mathematical (Solid State) Physics. However, "The Futur's Not Ours to See", you know !

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London Bridge is broken down,
Dance over my Lady lee ;
London Bridge is broken down,
With a gay lady.

How shall we build it up again ?
Dance over my Lady Lee ;
How shall we build it up again ?
With a gay lady. <Mother Goose>



Gustave D

Fig. 17 The man of La Mancha, and his troop.