NOTE

"L'Addition, s'il Vous Plaît !" (No. 2)
"Wannier, Who ?"

Hiizu FUJITA

CONTENT :

ABSTRACT

The point defects that dwelled in The Solid State Physics is cut out mathematically. The latent period of defects was unusually long. However, the painful operation was achieved by Doc. Labesgue The Knife with the help of SET and DISTRIBUTION. The Temporal Conclusion, which ended up with 4-items so far, would be quite welcome by people who wants to resume good health and beauty. —"Clear is beautiful"—

§1 Résumé of "Bloch, Who ?" (No. 1), and Introduction to"Wannier, Who ?" (No. 2)

Dr. Robert Bachrach, who is one of my old friend since I had been working at University of Illinois, stopped off Tokyo, on July 13, 1997. He was going to attend The International Conference on X-ray and Extreme Ultraviolet Lithography (XEL97). We met at Yokohama Grand Intercontinental Hotel, and fell to talking about our "WHAT'S UP ?" I showed him the galley proof of "My Latest Crazy Work".

"What's your conclusion ?" said Bob. He is a busy-busy Director in the busiest Company, in the busiest Part of the busiest State, in the busiest Country in the World. He is used to say, "It is always good to be in a hurry !" "OK ! I'll show you my conclusion !", I returned the ball into his court.

Reading the final section of the "Crazy Article," Bob said ; "You should not say crazy for yourself. Crazy is not a Good Word. If you say this work is Crazy, then nobody will read it. This is not a joke. Do you hear ? Do you hear ?"

Bob pointed out a few words, which should be corrected defnitly. So I jumped through a hoop. Bob's sermon continued ; "You need not Pretend to be Crazy, when you became as of your age ! You should mention your thought straight !" Oui ! J'ais soixante-cinq ans !

"Look at this ! I'm getting to be a Santa !" said I, when I met Dr. David Lynch, Iowa State University, at the SRC (Synchrotron Radiation Center ; Stoughton, Wisconsin), pointing my Salt-and-Black-Pepper eye brows. They had "SRC Rededication Ceremony ; Ednor M. Rowe Synchrotron Radiation Center, on May 3, 1997", and I joined it after these about 30 years.

"Yeah !" said Dave, "that is the way, the chemical process is going to fix !" Sure ! So now, who afraids of the Japanese Theorists, who have been opossing me ever since I was born ? I should get a move on to mention before the chemical process will stop ; well, we will see !

In the previous article, "Bloch, Who? (No. 1)", I pointed out three things ;

【A】 A set of Aleph-Zero cannot become a set of Aleph-One.

【B】 A continuous projection of a function f(x) on, or within, a 3-dimensional sphere B^3, has at least one Fixed-Point.

【C】 A projection of an open set, always ends up with an open set.

It may sounded like an Abstract Nonsense to the majority of readers, and

I added some yak-yak statements in the work. However, as Dr. Bachrach pointed out, I think I would better mention flatly, exactly, and precisely. I might add here some extra explanations on the above statements, 〖A〗, 〖B〗, and 〖C〗 :

〖A〗 Many textbook of Solid State Physics usually start in Counting Atoms along the line of the Crystal. They count atoms by Numbering them from 0 to N, or from 1 to N-1, as you see in Fig. 1 ;

Let's divide the each number by the maximum number, N, and re-lavel the figures as indicated in Fig. 1. Now, as you all know well, the numbers such as 0, 1/N, 2/N ····· 1-1/N, and 1, are called RATIONAL NUMBER. Cantor had already shown (in December 1873) that the SET (集合) of Rational Number belongs to ALEPH ZERO, just as the same as NATURAL NUMBER. 〖Ref. 17 : The reference list is shown at the end of the previous article No. 1 "Bloch, Who ?"〗 However, the point is here ; you will never be albe to fill up the line, however much you may try by letting N bigger and bigger, even up to infinity !

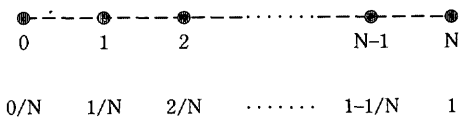

Fig. 1 A line of N atoms is supposed to make 1-dimensional crystal.

The majority of Theoretical Japanese Solid State Physicist (here after the abbreviated form "T-JSSP" will be used) say and write that we will get

to "Almost Continuous Limit", when we increase N. This statememt is a complete False. Because, again, Cantor showed the REAL NUMBER has ALEPH ONE infinity, while Rational Number is ALEPH ZERO infinity. So, you have the discontinuity a.e. [Almost Everywhere : this is a formal technical term (jargon) of Mathematics]. Acording to Cantor, we have formula as following 【Ref. 17】 ;

$$(\text{aleph zero}) + (\text{aleph zero}) + \cdots\cdots = (\text{aleph zero})$$
$$(\text{aleph zero}) \times (\text{aleoh zero}) \times \cdots\cdots = (\text{aleoh zero})$$

So, you are in a losing battle if you try to patch up HOLES by increasing N. I described this problem, by the word "Dr. Bloch's Leaky Table Cloth", in the previous article, "Bloch, Who ?" (No. 1).

I heard once, such a story as I mentioned above is well known even to The Elemental School Puplis in Switzerland. While in this country, T-JSSP are harping on the same string for about half a century.

【B】 The problem of Fixed-Point Theorem is stated in a decent mathematical way as follows 【Ref. 19】 :

(1) Definition of Fixed-Point :
Whenever, point $x \in X$ satisfys $f(x) = x$, under the projection of $f : X \to X$, then, x is called FIXED-POINT for f.

(2) Brower and Poincaré's Theorem :
Continuous projection $f : B^n \to B^n$, has (at least one) Fixed-Point.
(where B^n stands for n-dimentional "closed ball")

I should say, the meaning of the theorem is so serious for us!
Beginning from the conclusion, "The (so called) Bloch Function cannot exist at all, within n = 2, and 3-dimensional Euclidean Space!"

The item (1) defines the meaning of FIXED-POINT. If an element (point, for example) that belongs (∈) to a SET (集合) X is projected to the same SET X, with the condition that f(x) is continuous all the point within (the neighborhood of) X, then it is called CONTINUOUS PROJECTION.

The item (2) says when the continuous projection is performed with the n-dimensional (closed) BALL (B^n), then there must be more than one FIXED POINT.

The combination of the both (1) and (2), leads to the following conclusion ; "There is no steady flow, such as *The Bloch Function* which circulate from $-\infty$ to $+\infty$, either on X-, Y-, or Z-cordinate, as *The Running Wave*!" Since, 3-dimentional "Ball" is Topologically equal to the Cubic Crytal in Euclidean Space.

How come such a fiddle-stick could be happened? I don't know! But, I would point out just one thing :
It was in early 1895, when Henri Poincaré issued his first article "Analysis situs". The Complement came out on 1904. It was in 1921, when "Henri Poincaré in Memoriam" was published, after the World War 1. Then, there came a rush of 20th century Math, in 1920s.

Dr. Bloch's article came out in 1928, however, there is not a word printed on Topology. Incidentally, this work is not only to establish The Wave

Function but to figure out the extreme case of Tight Binding, Specific Heat of electrons, Acceleration, and Electron Interaction and Elastic Wave of Lattice. The first section where Dr. Bloch mentioned on the Wave Function is only 6 pages out of 45 pages ; this is a very comprehensive work.

It appears, I am afraid, none of the T-JSSP after the World War 2, consulted directly back into the original Dr. Bloch's Work.

When it comes to the relations between Henri Poincaré and Dr. Bloch (and with Heisenberg), probably there were still "Trench Effect", which separated two nations from Bergium to Swiss, by 400 km long trench. Well, I should refrain from talking too much about the Past that far. Incidentally, I've never seen the T-JSSP were talking of Lebesgue Integral in their SSP text-book.

As for the *Separation of Variables,* it is only known even as of today that the system of higher symmetry has a tendency to have higher *Integrability.* [Ref. 26] The Separation of Valiables sometimes ends up with to raise the symmetry. By this indirect effect it makes the system integrable. However, this is not always true, and this problem is not clearly solved yet.

Logically speaking, the origin of the disaster was in the negligence to keep weather eye open on Modern Math. The success of making a Ring out of a String was so Easily escalated up to 3-dimentional crystal.

As for the Interference of Bloch Waves, it is a sad business that even some Nobel Laurelites appeared to be not good at Classical Wave Optics. All the Professors' men follow, that no interference pattern shows up with

one slit. Actually, there comes out The Faunhofer Diffraction Patern, which is due to the Interference of Coherently Diffracted Wave, from only One Slit (or an Opening). What's more, there comes even a Single Edge Diffraction. Therefore, the point is, why waves do diffract after a slit? Double slit Fringe Patern is only the results of the interference due to diffracted waves from two adjacent slits.

It is nothing to do with the Game of "Appearance-Dis-appearance" of Wave and Particle property. As a matter of fact, dozen of optical systems were invented to get interference pattern without employing double slit system. [Ref. 27] This was because, the Young's fringe experiments were very hard to reproduce. This is So-Well-Known Fact for sincere Experimentalists of Optics. However, somehow or other, the Professors' men still publishing hell amount of text-books, which is no good at all for students.

This is one of the Why, I think I should cry ;

"Garçon! L'addition, s'il vous plaît!" ("Waiter! Gim'me a bill! I wanna get out!")

As the conclusion of §1, I should summarize it as following ;

(1) In No. 1, I pointed out the Bloch Function is false. It was derived upon the serious violations of the Topology and Modern Math. Theoretical Japanese Solid State Physicists should have realized this, if they were diligent enough to dig up and keep up the progress of modern French-American Mathematics.

(2) In No. 2, I will point out the Dr. Wannier is the second to Dr. Bloch. I will point out how many serious violations and negligences were on the air, with regard to Integration of the Functions. The 20th century is almost ending. Nevertheless, Theoretical Japanese Solid State Physicists (T-JSSP) appear to be drowsing before the entrance door of 21st century. Do we need another Admiral Perry's Black Ship Fleet, in Tokyo Bay, to let them awake? C'est possible!

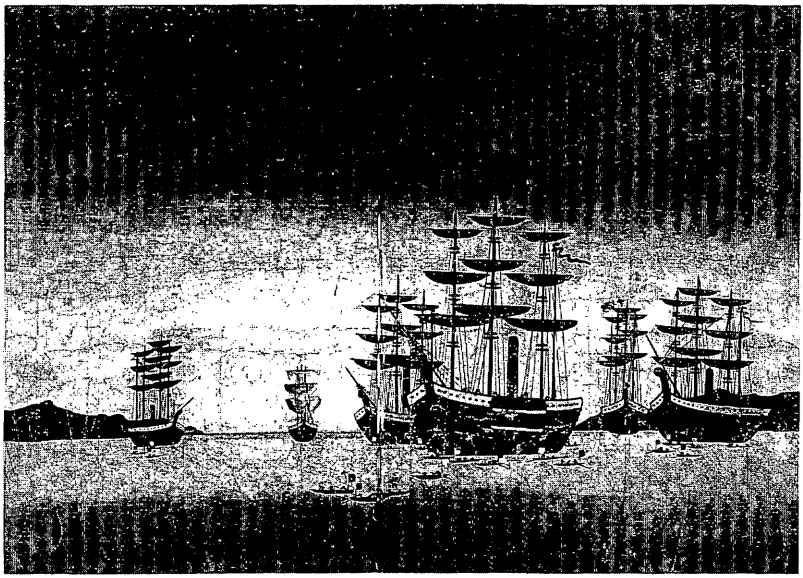

Fig. 2 Admiral Perry's "Black Ship" Fleet. (painted by Tika Haru, in, 1854)

§2 What Went Wrong, with Dr. Wannier?

Following the Dr. Bachrach's advice, I would use plain and clear words in this section. Another words, I will speak by the language of Mathematics. The text that I use for my talk is the Dr. Wannier's original paper [Ref. 28]. Now, let's open his article and see what did he say ;

In the abstruct, he says ; "a method is devised to study the energy spectrum for an excited electron configuration__a single excited electron taken out of a full band of N electrons__because of the Coulomb attraction between the electron and its hole $N^{8/5}$ states are split off from the bottom of the excited Bloch band ; for these states the electron cannot escape its hole completely."

Further, he says ;
"We shall restrict ourselves in this article to insulators containing one electron in the lowest excited state__As to the method we shall proceed in the following way : (1) We shall construct orthogonal "atomic" wave functions and express the energy matrix in this vector system."

"1. Basic Wave Functions and Energy Matrix"

"It would no doubt be more satisfactory for insulating crystals, to discuss the Hamiltonian using atomic functions rather than Bloch function."

__What does it mean by "would no donbt" ? Politeness ?
=Maybe.
__How come he can say, "more satisfactory～than Bloch function" ? Did he calculate ? And compared ? Or, just intuition ?
=Maybe, that was the Spirit of the ERA.
__Does it mean that he is saying "The solid state physics is the Local Physics" rather than Non-local one ?
=Well, I don't know! Anyway, he says "Bloch functin". Let's keep this in mind.

Dr. Wannier begins his great march ; "But this line of attack has been hampered by the fact that atomic functions are not orthogonal."

__I don't quite understand what does he mean by "Atomic functions"? Nobody has atomic functions except Hydrogen Atom, does he? You know, Helium Atom is not solved yet as of 1997 ; because it is the famous Three Body Problem. 【Ref. 29】

=No. He means a "Virtual" or an "Ideal", or would better say "Primitive", Atomic Functions, and an Array of them. More rigorously say, a Determinant of Undeterminable Elements!

__What's that?

= Well, you may know, that Dr. Heisenberg's Final Understanding of the Universe was that, the most fundamental thing is the Symmetry of Particle Functions! This idea became the Spirit of the ERA. You will find it everywhere if you want.

__Then, what's for "not orthogonal"?

=To my opinion, that's just to scare peoples. Or, if I should be more polite, he says, "I'm not forgetting the many-body problem". What's more, he means the way he devised is the Orthogonal Function to N^3 different atom's Atomic Functions!

__Wow!

=Do you happen to enjoy Poker Game?

__No. I guess it's a Wild Wild Game.

=Too bad! This is a bluff of the game! However, as you've read my article No. 1, I hope, you will enjoy this article, No. 2. You may find youself as if you are reading a detective story from the end to the front. Since you knew there's no Bloch function in 3-dimensional Euclidean Space, any more.

Dr. Wannier beats the drum ;

"We can, however, build up orthogonal functions having all the advantages of atomic ones by starting out from a Bloch approximation."

=You will soon find the "atomic one" is a real monster. If you are the atomic physicist you would completely agree with me. I guess I'd better leave this subject until later occations. However, I'd just give you a warning so that you would not be surprised to know that what a strange thing the Solid State Physicists (SSP) are admiring!

__Then, what does it mean by "*all the advantages* of atomic ones" ?

=That's exactly what I'm saying ; they (SSP) think, or believe, that their "atomic Function" is a real beauty!

__But, what's the "Bloch approximation" ? He said "Bloch function", didn't he ?

=Yeah! That's another puzzling thing for me either! You'll soon realize, there is no well settled terminology, or "jargon" in this community. Different shool has its own language, to my opinion.

__Wild-Wild-West, ahem ?

=Not exactly. To begin with, you must well recognize that we are talking about very-very old subject. The Average American Physicists are sure to hear that we are talking of "The Useless Antique" (Les Curiosités), such as Lincoln's Baby Shoes. The year of 1937 is so ancient for Americans. It must sound like as if we are talking about the 1st Christmas Eve of the Human History!

=Now, let me consult my book in my study, and here we go! Dr. Muto says in his monumental lecture note [Ref. 30] on "The Bloch Sum Approxima-

tion" :

"When potential V(r) is given, it becomes necessary to calculate the Bloch Orbital Function, and the corresponding energy band structure. The Approximation that I am going to talk is "The Method of Bloch Sum, or The Tight Binding *Approximation,* which has a correspondence to The LCAO (Linear Combination of Atomic Orbitals) Method for the Molecular structure theory."

= Dr. Muto shows the "Bloch Orbital Function", in the section, named "Bloch Sum Method *Approximation,* or LCAO Method" (in §4 · 5 · 2), as following ;

$$\Psi_k(r) = \sum_{R_n} C(R_n)\,\phi_s(r-R_n) \quad \cdots\cdots (4 \cdot 111)$$

where, $C(R_n) = Ne^{ikR_n}$ $\cdots\cdots (4 \cdot 113)$

and combining eqs. (4 · 111) and (4 · 113), we get ;

$$\Psi_k(r) = N \sum_{R_n} e^{ikR_n}\,\phi_s(r - R_n) \quad \cdots\cdots (3\text{-}1)$$

= Dr. Muto also says in §4 · 5 · 1 ; the title of which is "General Features of Bloch Orbital Function and Wannier Function", refering to G. H. Wannier's work, Phys. Rev. 52, 191, (1937), which is exactly that we are reading now. He says ;

"Therefore, we get as the result of the reverse Fourier Transformatation, the following equation,

$$\Psi_{n,k}(r) = G^{-3/2} \sum_{R_j} e^{ikR_j} a_n (r - R_j)". \quad \cdots\cdots (4 \cdot 102)$$

= By comparison, eq (3-1) with (4 · 102), you see that Wannier function is formally equal to the "Atomic Function" in LCAO, or Tight Binding Approximation. Of course, you can set The Wannier Function to the left side of equal (=), by reversing the eq. (4 · 102). Then Wannier function, a_n, is composed by the linear combination of Bloch function. At any rate, they (Atomic functions and Wannier functions) are inter-dependent bases in a Function vecter space for the Bloch Sum Function.

= Incidentally, I heard funny stories on Wannier function, more than dozen times, that the Wannier function had succeeded in to Localize the Bloch's spreading function. This is just a confusion between Bloch's (Original) function and Bloch Sum (Oribital) function. Bloch Sum function must had been localized, if the LCAO or Tight Binding Approximation have ended up with success for Localization of the Running Wave ; as (chemical) people claims so.

= Of course, Dr. Wannier must had thought that his Function is better, or more fundamental, than Bloch's Original function. However, here goes interesting "Game of Cards". Watch it very carefully. How he is good at to "Schuffle the Cards" ! This is "The State of Art".

= Actually, Dr. Wannier starts from the reverse transformation of eq. (4 · 102). However, I would leave this old, nasty, musty calculations to readers. I would like to talk more important NEW subjects, in this work.

Dr. Wannier proceeds further ;

"Let us *assume* then that a Bloch or Fock method has given us functions b (k, x) of energy W (k). Then *required* funcitons are,

$$a(x - n) = 1/(N)^{1/2} \cdot \sum_{\nu} \exp[-ik_{\nu}n]\, b(k_{\nu}, x)^3, \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (1)$$

where N is the number of cells in the crystal and the k's are as usual determined by some *boundary condition.*"

_How come Dr. Wannier says *ASSUME*? It sounds like impolite for Dr. Bloch, somehow. You know, Dr. Bloch's paper appeared in 1929, and it is 1937. Eight years have past, with good reputation, I'd imagine.

=Well, the year of 1937 is not quite simple. In January 31, 1933, Hitler grabbed the Political Power of Government, after the National Vote. In 1936, Italy took over Ethiopia, in Africa, by Force. In the same year, Japan, Germany, and Italy made the Three States Axial Union Treaty, under the name of Anti-Communism. Actually, in September 1, 1939, German Forces crossed the border to Poland.

=In these days, in 1934, Dr. Bloch moved in to US. Dr. Bloch was a Swiss-American. He was born in 1905. He graduated from Zürig University (?) and he got his PhD at Leipzig ; probably by the work that we have read in "Boch, Who?" (No. 1). 【Ref. 6】 Dr. Bloch moved in to US, and he joined the Manhattan Project, later. 【Ref. 31】 So, Dr. Bloch must had said a "Farewell to his《herzlichsten ge-Dankenen Herrn Professor Heisenberg》" (heartily thanked Prof. Heizenberg!)

=Dr. Wannier was in Princeton, New Jersey. So, it's very possible that he

had complexity towards Dr. Bloch ; whether he should take him as a friend or rival. I really don't know, either the word "assume" has a sense of suspicion, or just for rhetoric.

__What about "required function" ? Who required ? Do "they" required ?
=Well, I guess there're long story behind it. Why don't we discuss later ?

◆◆COMMENTS

__Now, I would like to make comments on "some boundary condition". As I have pointed out, rather strongly, in No. 1 article, if you would get a "Steady Running Wave in a 3-Dimentional *Euclidean Space*" via "ANY Boundary Condition", Topology is sure to put the Veto on your conclusion. However, there are 2-choices that the Solid State Physics can survive within The Euclidean Space ;

1) Approve that there is only 1-dimentional solution, if you want to stick to the Bloch Function ; which stretches from $-\infty$ to ∞. This choice ends up with the Corollary that all the phenomena we should be able to find in Solid is 1-dimentional phenomena, only !

2) Approve that the Solid State Physics has only "Local-World" to play with any sort of Current or Waves to run ; by "Local-World", I mean the axes $X-$, $Y-$, and $Z-$ cannot be stretched from $-\infty$ to ∞. Mathematically, we can have only *Closed Set* of Real Number Axis. Physically speaking, the Source and Sink Terms break in always, whenever you speak about Flow-Dynamics.

= To my imagination concern, there is no other way to escape. On the

other hand, if you give up to stick to the "3-dimensional Linear STEADY FLOW", then you can play with any Dynamics ; either in a TORUS or Euclidean World. In this case, however, you should admit that unpredictable number of FIXED POINTS may break-in (possibly with Chaos). This is the very common thing in Meteorology.

=Now, let's continue to hear what Dr. Wannier says ;

"Formula (1) applies to any *set* of Bloch functions, *but* it might be interesting to get some insight into the structure of the a's. For *this* purpose let us first make the *ad hoc* assumption (valid for *free electrons*) that b is of the form

$b(k_\nu, x) = \exp[ik_\nu x] \cdot b(x)$, ... [eq. A] (let's call this as [eq. A])

where the periodic factor b(x) is independent of k."

=Well, did you see it ? Here are so many things. Maybe you've missed all !

(1) What does it mean by "set" of Bloch function ? The word "set" means, if it was used in mathematics, an agregation (die Menge, 集合) of "Elements". If it was used, however, at a coeffee counter, like "Morning Set", then it means, "Toast-N-Coffee, with 2-eggs, choice of Scramble or Sunny Side Up ?"

This is not a joke. If Dr. Wannier means "A Set of Function", then you got to learn many things beforehand. Probably you know quite well about the Number Axis, and also Number Vector Space (sure !).

However, when it comes to a "Functional Vector Space", and/or "Set of Function" on an Axis, then this is something! It is just a step to go upward for someone, but it can be a desperate Overhang for another ; formidable for him to climb up!

_Which do you think this SET is?

=I don't know! Dr. Wannier is surely not talking of Math. Otherwise, he should had found himself colliding with Bloch function through Topology, as we did.

(2) In the middle of the section, Dr. Wannier says, "*but* it might be interesting……". (See where it is?) As far as I know, the word "but" is a logical word. Normally, the word "but" denys the statement ahead of it, and it rolls out a statement which has opposite logical meaning. Then what Dr. Wannier is saying "but it might be interesting……"? Is it not interesting for him "Formula (1) applies to any set"?

=I don't quite understand either.

=So, sometimes, old folks say, "You cannot start a sentence by But!" I still remember the most funny lecture I heard. The professor started his lecture one day, saying, "Nevertheless! ……"(とは言うものの!) He was obviously figuring the story for the lecture, all the way through the hall to the class-room.

=Incidentally, the professor was a very logical person. Sometimes, he performed the talk without intermission at all. It sounded like as if we were cruising down a stream by a tiny ship. However sometimes, he ended

with a sharp nail for such a smooth lecture, by saying "*But, this* is not true!" (〜ではない!)

=We were all at a loss. "What did he mean by This? From the very beginning of *this* lecture? That's an awful tragedy, if so!"

=Well, I hope, Dr. Wannier is sort of a man like the professor. Otherwise, *this* is a disaster, you know!

(3) How come Dr. Wannier says, "let us make the *ad hoc* assumption (valid for free electron) that b is of the form [eq. A]……"? My compromise for this problem is that, "People of the ERA may meant【Electrons in Metal】by the word【free electron】". If so, ;

(3-1) Did Dr. Bloch say that he was handling the problem of Metal? Where is the *ad hoc* committee that approves "*ad hoc* assumption"? Did thet agree that the Bloch function [eq. A] was used to free electronsy?

(3-2) What Dr. Wannier is going to say, by employing (metal) free electron for this article, which is clearly titled "Insulating Crystals"?

_He did it again! We got to go back to Dr. Bloch's original paper, and got to look at whether Dr. Bloch said he was handling free electron.

=Don't waste our time. I know quite well. Dr. Bloch examined free electron case in §1, by crossing out the potential term, V, in the Differential Equation. He didn't call it as Schrödinger's equation, by the way.

=Dr. Bloch started from,

$$\Delta\psi + \mu (E - V) \psi = 0,$$

and got to the "bekanten ebenen de Broglie-Wellen", that is

$$\psi = 1/\sqrt{KLM} \cdot e^{2\pi i (kx/k + ly/L + mZ/M)}$$

__Quite simple. Very good.

= And in §2, Dr. Bloch handels the "stark gebundenen Elektronen" (tight binding electron). I don't think this is free electron case.

__Ok. Let's see what Dr. Wannier says in the following.

=Dr. Wannier says ;

"Then we find *explicitly* :

$$a (x - n) = \frac{\sin \pi (x_1 - n_1) \sin \pi (x_2 - n_2) \sin \pi (x_3 - n_3)}{\pi^3 (x_1 - n_1)(x_2 - n_2)(x_3 - n_3)} \cdot b (x) \cdots\cdots \text{[eq. B]}$$

which gives us the desired concentration around n_1, n_2, n_3."

〖STOP〗

=Clear ! We'd better get out of here, Leave your blue chips on the table. Forget it !

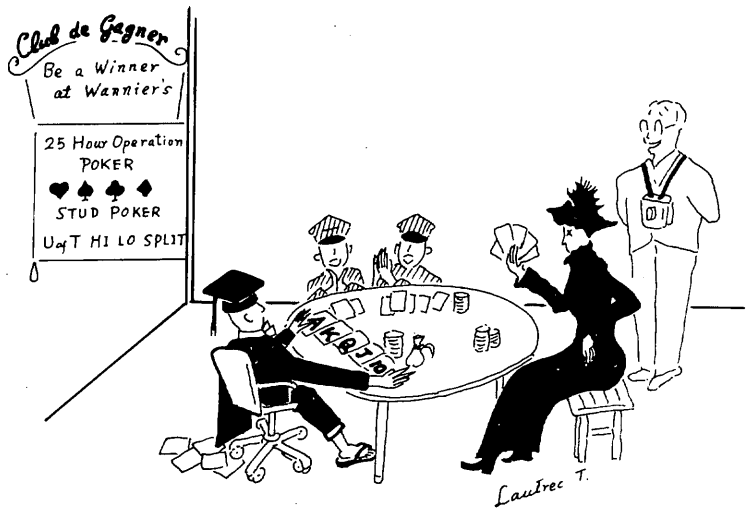

Fig. 3 Dr. Wannier's Loyal Straight Flash hand, at "Club de GAGNER" (Refered to Lautrec)

§3 I'm telling you !

◆**3-1.** First of all, you cannot get to [eq. B] *explicitly* from [eq. A] ; never ! It is just a matter of Geometric Series Sum calculation. So, even if you are a junior-high school boy/girl, you can manipulate it. Of course, you are supposed to know a little trick to divide the argument of exponential factor into half ; so that you can get to $(\sin x)/x$.

However, even though you were a professor, you will never get to [eq. B], so long as you stick to Summation ($\sum$) ; either from 0 to N, or from $-1/2N$ to $+1/2N$. You will soon convince yourself that a factor of complex exponential is sticking around you, and never let you go. Here, "N is the number of cells in the crystal", of course. You've "read and understood it" in the text, don't you ?

The only promising way to get to [eq. B] from [eq. A] is to sneak away from sigma ($\sum$), and grab of integral ($\int$) as following ;

We have, $a(x-n) = 1/(N)^{1/2} \cdot \sum_{\nu} \exp[-ik_\nu n]\ b(k_\nu, x)$ ····· (1)

where $b(k_\nu, x) = \exp[ik_\nu x] \cdot b(x)$. ····· [eq. A]

By combining the two equations, we get,

$$a(x-n) = 1/(N)^{1/2} \cdot \sum_{\nu} \exp[ik_\nu (x-n)]\ b(x)$$

Then, stretch $\sum$ and deform it to $\int$, and paint the lily as following [Ref. 32] ;

"There is a *famous definition* of δ-function via Integral Form : i.e.

$$\int_{-a}^{a} e^{iwt}\, d\omega = [e^{iwt}/it]_{-a}^{a} = (2/t)[(e^{iat} - e^{-iat})/2i] = 2/t \cdot \sin(at)$$

$$\therefore \frac{\sin(at)}{\pi t} = (1/2\pi) \int_{-a}^{a} e^{iwt}\, d\omega$$

Putting $a \to \infty$, we get to $\delta(t) = \lim_{a\to\infty} (1/2\pi) \int_{-a}^{a} e^{iwt}\, d\omega$

$$= (1/2\pi) \int_{-\infty}^{\infty} e^{iwt}\, d\omega.$$

Therefore, by setting $a = \pi$, we get to the Wannier's form [eq. B],

$$a(x-n) = \frac{\sin\pi(x_1 - n_1)\sin\pi(x_2 - n_2)\sin\pi(x_3 - n_3)}{\pi^3 (x_1 - n_1)(x_2 - n_2)(x_3 - n_3)} \cdot b(x).\text{"} \quad \cdots [\text{eq. B}]$$

◆◆**3-2.** All that stated above is nothing to do with the Modern Math. These are the Classical Math. I would say, 19th Century's mathematics.

The first thing I must point out here is "You Cannot Integrate over k-axis". The reason is simple ; the function you are handling is not defined all over the k-axis. I can point out thousands of text books, which are wisely say that the Bloch Function is defined only at, or upon, the mesh-point in k-space (cf. "Bloch, Who?"). Some (or maybe all) Theoretical JSSP should cry out. "We know such a thing 100 by 100% sure !" (重々承知だ !). But actually, they don't know anything.

A friend of mine pointed out that Dr. Ziman says in his book【Ref. 33】; "In practice, *N is very large,* so that this distribution is treated as *continuous.* We often express a sum over k-vector as an integral

$$\sum_{k} \rightarrow \int dk \equiv (V/8\pi^3)\iiint d^3k$$

using the single integral as a concise notation for the limit of them."

It will be interesting to dig up who was the first that made up such a story. However, the kindest friend of mine should be about to say ; "You should admit the rest of the world's consensus ! Look! Such a Famous Proffessor as Dr. Ziman in Cambridge is stating this way ! 【You are the stubborn idiot, Doc. Fujita !】".

It is amazing to notice that Lebesgue, Henri Léon (1875-1941) invented "The Lebesgue Integral" around 1910. Nevertheless, No Theoretical SSP ever faced to it. They just continued to fool The Modern Math, and dis-

graced The Solid State Physics for more than three quarter-th of a century !

Another friend of mine says in his text book 【Ref. 34】 :

"Since N_i is a sufficiently large integer, the value of k changes almost continuously, and moreover, *countable*-way (可付番的に) ; where k is defined by,

$$k = \sum_{i=1}^{3} (m_i/N_i)\, b_i."$$

(b is the basis of reciplocal lattice vector to the Bravais lattice)

= Well, he is honest, but he may not know what he really wants to say : There is a Theorem, on function analysis 【Ref. 35】, i.e. "A *continuous* function which is defined upon a closed set, [a, b], is *integrable*". This is it ! A big chorus follows. "Bloch function is *almost-continuous,* therefore it is Integrable !".

◆◆◆**3-3.** You are all in a box !

Let me show you an example : Suppose we have an array of posts, as shown in Fig. 4. You want to integrate, somehow or other, the total area that the posts occupys !(?)

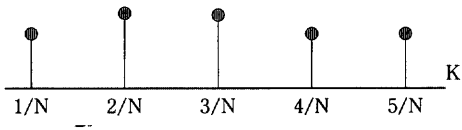

Fig. 4 A winding array of N points in k-space. Someone wants to integrate, somehow, the area covered by the posts, as shown in the Figure 4.

Table 1 TABLE OF RATIONAL NUMBERS (m/N)

m \ N	1	2	3	4	5	6	7	8	9	10	···
1	1/1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10	···
	1.0	.5	.33	.25	.2	.166	.142	.125	.111	.1	
2	2/1	2/2	2/3	2/4	2/5	2/6	2/7	2/8	2/9	2/10	···
			.66	.5	.4	.333	.285	.25	.222	.2	
3	3/1	3/2	3/3	3/4	3/5	3/6	3/7	3/8	3/9	3/10	···
				.75	.6	.5	.428	.375	.333	.3	

♣ **3-3-1.** Let's examine first; "How continuously k changes when N is sufficiently large?" Of course you know well that I can call the figures (m/N) as Rational Number. Incidentally. Dr. Bloch also says in his article that "und k, l, m ganze Zahlen bedeuten (k, l, m mean integers)". The fraction of the two Integers make a Rational Number (有理数). It becomes a recurring (repeating) decimal (循環小数), when you cannot finish deviding. Why don't you look at the Table 1;

Let's think about when $N = 10$; although this is not "sufficiently large" number. In this case, the k-axis in Fig. 4 has marks only 1/10, 2/10, 3/10, 4/10, ·····. Then, how many rational numbers you have skipped between 1/10 and 2/10? Look at the Table 1; four figures. They are from .111 through .166. Then there comes .2, which is equal to 2/10 on the second row. Now you see, the rational numbers before the middle point on the first row, have been all skipped.

So when you go up to "sufficiently large integer", the number of skipped figures increase astronomically! When, say $N = 1 \times 10^7$, (cf. No. 1; it is estimated by putting $L = 1$ cm and the lattice parameter $= 1 \times 10^{-7}$ cm) the number of skipped figures go up to $1/2 \times 10^7$. Very ironical! What's

more, the figures below the middle point, such as 0.25, comes back to the skipped number, later in 2nd row. These are the points where the function is not defined upon rational number set.

Therefore, from the point of view of the "Set of Rational Number", there are so many point of discontinuity on k-axis associated with the Bloch function.

Of course, "The Clever Boys" reply immediately ; "We have Real Number Axis in k-space from the Beginning ! The points you are talking is just Eigen-Values for the Schrödinger Equation. Ha, ha, ha !" (You will shed tears, pri-soon ! Ho, ho, ho !)

♣♣3-3-2. Secondly, The Riemann (definit) Integral has definit value, when the following necessary-and-sufficient condition is fulfiled : "The function must be continuous *a.e.* (*almost everywhere*)". Yeah, you may insist so upon this function, but I won't join with you. The way you are doing is just to disgrace the Solid State Physics, from the Theoretical side. The reason is simple : You are just too faithful to Riemann 〖Georg Friedrich Bernhard (1826-1866)〗 Integral. Surely, Dr. Riemann must be a big daddy in mathematics. However, he may had read some News about President Lincoln at a breakfast table, if he had habit to open newspapers !

Tell you what, why don't we consult with Monsieur Lebesgue 〖Ref. 17〗 about such a hanky-panky game ? I don't know whether he read the News on Pearl-Harbor, but he must had heard Hitler and de Gaulle's address. Sounds better, isn't it ?

In order to study "the Lebesgue Integral", readers are recommended to learn by yourself. It is considered absolute necessity to make himself mas-

ter of Math that the student must feel eagerness to study mathematics, first of all. Another words, he must find by himself "Will to Learn".

Then the next necessity is "Will to Do-It-Yourself". This is because Math can never been understood only by teaching. Good teacher, if not a professor, can give you a fragments of idea. However, it won't deposit nor crystallize as your knowledge without your "deep concentration" 【Ref. 26】.

If you would allow me to make an excursion, the same idea i.e., "How nice and important it is to study by yourself !." is aleady writen by the beautiful composition in "Souvenirs Entomologiques. 1879-1910" (Souvenir of Entomology. 1879-1910 : ファーブル『昆虫記』) by Jean Henri Fabre ; Vol. 9, Chap. 13, 14.——"Retrospect on Mathematics". Very fortunately, a complete set of Japanese translation is commercially available. You will find J. H. Fabre was not a simple bug-hunter.

♣♣♣**3-3-3.** Alas ! Your Lebesgue measure is ZERO ! (御愁傷様 !)

First of all, let me remind you that in 【Ref. 34】 the author of the text book left a few words ; "almost continuously, and moreover, *countable*-way ; ". Readers may be able to swallow it, but should be hard to digest it. Obviously, the writer may have a few teeth to bite a li'll-bit of modern math, or he may have a few bad friends. Since, he got "A well begun, but only half done" !

In an introductory book on Lebesgue integral 【Ref. 17】, you'll soon find a statement ; "Countable set of *point* is Measurable" (on Lebesgue integral scheme). However, (the writer of the text book should have read the next line), "and its (point's) Lebesgue measure is ZERO" !

Unhappiness comes never be alone. On the following paragraph, the writer should have also read, "The set of *Rational Number* is measurable, and its length, i.e., Lebesgue measure is ZERO"! Now you may smell the smoke of gun powder. Yeah. There will be so many that injoured seriously, I hope !

Let me give you a combat field instruction, how to use our weapon, i.e., The Lebesgue Integral. You may have half realized, that "The Solid State Physics" is turning out to be much more sophisticated Science than Peoples have ever been thought of !

♥Quickly speaking, The Lebesgue Integral is determined by two important factors :

1) The character of the function itself.
2) The character of, what you call, the Axis. More rigorously say, the SET upon which the function is defined.

The item 2) was not taken into considerstions with sufficient care, in Riemann integral. Another words, it is a cut and dried calculation. I should explain more definitly later, using mathematical words. At any rate, we must examine first, with a good care, on what axis (SET) the functions are standing on. By so doing, we should be able to Integrate, even taking discontinuity into account.

♥♥To begin with, let me speak of the item 2). Since this is the New Thing, and unfamiliar for someone. It's too bad for Japanese Hi-school students that No Teacher wants even to introduce the 20th century's integral.

Actually, the concept of "Lebesgue Measure" is not so easy thing to swal-

low. However, once it was swallowed, it must be easy to digest for young people. No one these days, for example, worries about how to verify the Pythagorean Theorem before he orders a kit for roofing. None of the research staff reminds of the lengthy slicing business of a sphere before he estimate the volume of a ball. Thus should be the Lebesgue Measure. We got to be more familiar with the 20th century's Modern idea. Gentlemen, Teachers, Professors, Scholars, Minister of Education, President of the United States !, 20th century is almost running away from us !

♥♥♥In short, we needed the Width (dx) so that we can estimate the Area (dS) by way of multiplying the Hight (I) times Width (dx), i.e., $dS = I \cdot dx$. We need this time a "sophisticated width", since we want to Take-In the point of discontinuity into our country. This "width" is, roughly speaking, the Lebesgue Measure, m(E). For the most simplest example, let me show you Fig. 5 :

The way it goes is as following ,

$$\alpha \leqq f(t) \leqq \beta$$

$$\therefore \quad \alpha \cdot m([0,1]) \leqq \int_{[0,1]} f(t)dt \leqq \beta \cdot m([0,1])$$

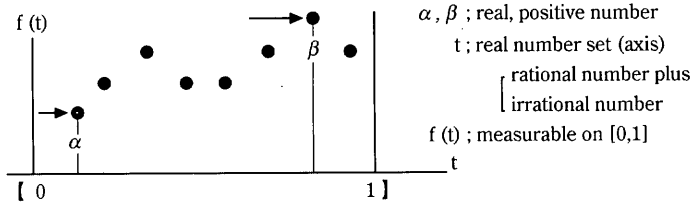

Fig. 5 The most simple example of "How to do it with Lebesgue Integral".

Where, m ([0, 1]) is the Lebesgue measure, and it is the "Length of Set [0, 1]" ; when the AXIS [0, 1] is the Real Number. Therefore, $\alpha \cdot$ m ([0, 1]) represents the Smallest Area size ; while $\beta \cdot$ m ([0, 1]) means the Largest Area size.

As you see, the difference of Notation for Intergration is very small. The integration mark ($\int$) is associated with just a tiny suffix [0, 1]. But this small suffix symbolizes that this integration mark carrys BIG different Meaning and Method from the popular Riemann integral.

The notation [0, 1] means that the "Measurable SET of Number" within/upon the "Closed Set, from 0 to 1" should be taken into account for the integral. Another words, we got to double check, point by point, dx by dx, along the Line from 0 to 1, whether it is the "Qualified Point" as the representative to be added into the integral. However, the reality is rather simple ; there are only Rational and Irrational Numbers within the Real Number Set.

Now, here break-in The Set Theory. The theory says, you can separate the set into Sum of Subset. Of course, Separate but with Equal rights and significance. In our case, the set [0, 1] is composed by real number, i.e., the sum of rational number and irrational number. Don't be astonished to hear, that the numbers of irrational number within the set is infinity. However, the numbers of rational number is also infinity, "But it is Countable (!)" as the writer of the text book [Ref. 34] has wisely pointed out. Here is the breakthrough!

The countable infinite set is Measurable! And, as I have already pointed out, the Lebesgue measure of Rational Number turned out to be zero.

While, the measure for real number [0, 1] is [1] ; the distance on real axis has no discontinuity. Therefore, the measure for our integration is also [1], since the "left over" taken away by the rational number measure is, $1 - 0 = 1$.

Now you can forget with ease, the suffix [0, 1] and the rational number points on the axis t ; since their Lebesgue measure is zero and there is no contribution from the rational number subset, whatever the value of f(t) maybe. Of course, we can forget abont the points if discontinuities appeared on the real axis ; since this time, there are no contribution from f(t) itself, i. e., there are no value on the function f(t) itself !

This situation is stated by the different words ;

Rational Number has Aleph Zero infinity, and Real Number has Aleph One infinity ; which is the "Density of Continuity". However much you may pile up the points of Aleph zero, say even up to infinity, you can never be able to get to Aleph one.

As you see, the Dr. Riemann's integral is fundamentaly taken-in by M. Lebesgue's integral. It is the author's feeling that this problem, "We cannot get to the Continume from Discrete State so easily, as we thought before", will raise serious troubles in every directions in Solid State Physics ; especially wherever Integrals should be taken into account.

§4 Calling a spade a spade

♠Professors of JSSP maybe reluctant to hear any more about their consen-

sus. However, so long as the group of "Physics History" has no ability at all to handle the modern physics, I have to leave some words to reveal the Spade-game.

The game goes as following, shown in Fig. 6 ;

(1) They believe in that the axis (x) is continuous (Real Number). They declare that "This is the nature of Nature!" or "Nature hates discontinuity!" So, they have thought there is no problem to set "width" (dx) on the axis, arbitrarily, which is inebitable for Reimann integral.

(2) Thus, they are unhappy to see the isolated (dis-continuous) values upon the real axis. So, they invented to do a "squeeze-game" by making N "sufficiently large". As the matter of result, they draw a curve [A] which connect from point to point. However, they dislike to employ such a line as [B], a chaine-line for example. Since it is not SMOOTH enough. It is

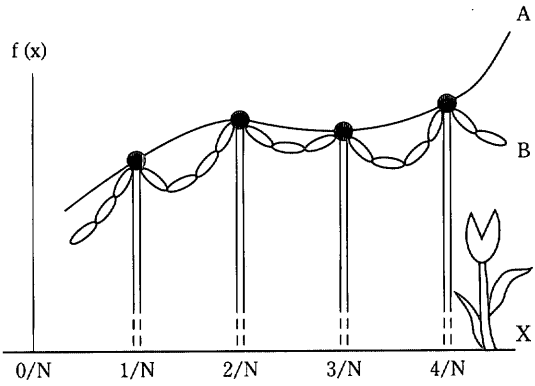

Fig. 6 How to create the nature by Riemann Integral.

their belief that the nature has the Smooth character, not only continuous, somehow. They don't know exactly what they are saying by the word "smooth". It means they prefer "The function of class C^1 or C^2". They whistle in the dark, "We have no trouble at all so far !"

By so doing, they have invented the nature. This is not modifying the Experimental Data. They are modifying their own Theory, and they believe it is Nature itself. So, in turn, they created the nature !

The optimist fell ten stories.
At each window bar
He shouted to his friends ;
"All right so far."
UNKNOWN

As the final message in this section ◆◆◆**3-3.**, I should point out another thing ; i.e., the function that Dr. Wannier showed in 【Ref. 28】 as [eq. B] is *incomplete.*

Surely, they are orthogonal and can be orthonormal as Dr. Wannier showed, but they are incomplete. Another words, they have no limit within the function set. I realy don't know what they were doing with such a function, even in Quantum Mechanics. People of mathematics say that "No Analysis can be achieved, with employing such functions as these their Limit would suddenly disappear!" 【Ref. 36】 However, this is just a minor subject for me right now. I want fly farther, not to say Fly High. I must save

ammunition and fuel for the farther ahead targets.

I might come back again here, maybe sometime, but so far it is none of my funeral. (May be it's Yours !)

§5 C^0s are coming ! Digitals to follow ! And another !

In the last section of the spade game, I pointed out that T-JSSP's most favorite thing is the Smooth Curve. They cherish smoothness for everything. They hate and avoid any singularity, and turn their heels and run away once they felt even the slightest mosquito-vibration of turn over. I know some friends of Dept. of Math., who complain often that the commercially available software always connects the discrete points by a smooth curve, where it should not be.

Probably, the world Physicists believe in that the Universe is made of C^1 or higher class functions. However, the cover up is getting over. Chaos is the roaring thunder. The Solid State Physics can't be the exceptional sanctuaries.

Curiously enough, mathematicians also didn't feel warm friendship to C^0 class functions. Someone says, "The function just continuous is awfully violent as rodeo." Let me interprete the languages that they are speaking :

"The function of class C^n " means that "A function f which is defined upon (a set) S, is *continuous* at any point on S, and it is differentiable up to n-th order. What's more, the resulted derivative function must be also *continuous.*"

Readers must be the eagle-eyed. It *doesn't* say that n-th derivative function must be also differentiable. Another words, resulted n-th derivative function is no longer differentiable. This is the end of your happy hunting ground.

Therefore, the function of "class C^0" means, it is continuous but non differentiable at any point even once! Sounds awful, isn't it? The most famous example is the "Koch triadic curve", as shown in Fig. 7, 【Ref. 11, 37】;

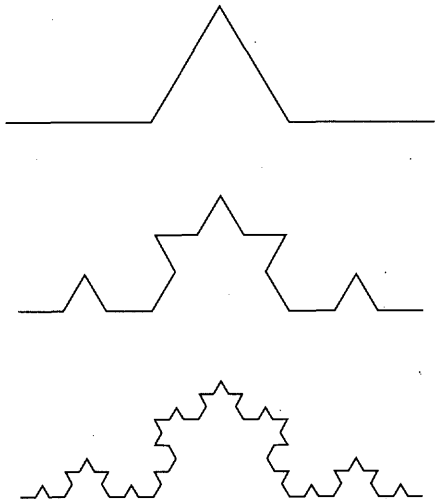

Fig. 7 A few example of Koch triadic curve.

The famous function that is non-differentiable, and as violent as rodeo is, The "Dirichlet function";

$$f(t) = \begin{cases} 1 \text{ (t : rational number point)} \\ 0 \text{ (t : irrational number point).} \end{cases}$$

The analytical representation of this function is the "Baire function" as following [Ref. 17] ;

$$f(t) = \lim_{m\to\infty} \left[\lim_{n\to\infty} \{ \cos(m!\,\pi t) \}^{2n} \right],$$

which is non-integrable by Riemann integral. I'm not sure at all whether the Baire functions can have the Inner product and/or Norm, so that they may be contained within the frame work of Hilbert Space : L^2 for the Schrödinger representation, and 1^2 for Haisenberg representation. [Ref. 17]

As for the isolated functions, or distribution, the famous ones are $\delta(t)$ function, and Channon's sampling function. These are inevitable for the digital engineerings such as Optic fiber and/or Disc business. Actually, what we call, "Sampling Theorem" is not at all new. It is a respectably classical Theorem related with the data handling theory for classical statistics. Maybe it's too old, and peoples almost forgot about it, I'm sfraid. [Ref. 38, 39]

This is the simplest application of Fourier Analysis, however, it's very useful to understand why we can cut off above a definite cut off frequency ; for example for digital recording.

The sampling theorem says as shown in Fig. 8 ;

"If you are sure that there is no frequency component (voices, for example) higher than W (Hertz), then the function f(t) can be reproduced precisely only by the data taken at every Δt interval, i.e.,

$\Delta t = 1/2\,W$ (sec).

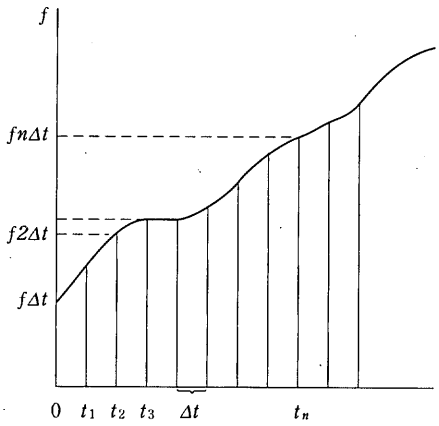

Fig. 8 An old example that shows the "sampling theorem". (Ref. 39)

This theorem works vice versa. If you have point-by-point data with interval of Δt, then you cannot say anything for sure above $W = 1/(2\Delta t)$. Therefore, there is very high possibility that the information higher than W is escaped from your system.

You may say you can extend k to infinity out side of the 1st Brillouin Zone. That doesn't make sense at all. As I told you, if you increase the number N up to infinity, that means you get infinite number of discontinuity even at the starting point. I will come back this point later in §7, by using more mathematical language.

However, real Hydro-dynamics is much more complicated than Math. Actually, even in the "art of India (chinese)-ink in water", there appears not only "Area Preserving Map". The maps such as, what I want to call, "Area Shrinking Map" and "Area Disappearing Map", show up easily. [Ref. 40,

Fig. 9-A

Fig. 9-B-1

Fig. 9-B-2

Fig. 9-B-3

Fig. 9 Examples of "Japanese Art of Marbling" (Suminagashi : 墨流し. Ref. 41, 42)
Fig. 9-A A private communication
Fig. 9-B-1, -2, -3, are expanded examples for the same region.

41, 42, 43,] [Fig. 9] Private Communication.

In these case, the stream-lines disappear after very complex motion near the Fixed Point. Sometimes it ends up with dotted line. If such phenomena were appeared in Solid State Physics, in AKP (Anisotropic Kepler Problem) for example, [Ref. 4], T-SSP would at a loss how to deal with it. I would imagine it would be better to think about it beforehand, how to do it with such Rodep functions.

§6 The Past Glories

Readers may wonder if the statement of this arrticle is true. Because there are obviously a great many historical success in Solid State Physics ; what they call, "Past Glories". Let me explain how the "Past glories" don't conflict with this article :

No one would deny that the greatest success in Solid State Physics is achieved by Semiconductor, or Transister Science. In §4 in the provious article "Bloch, Who?", I pointed out by citing the Dr. Shockley's book, "Electrons and Holes in Semiconductors" [Ref. 12]. At the end of the section 5.3, p134, he says ;

"However, our main interest in quantum-mechanical theory centers about the problems of electrical conductivity rather than those of cohesion". To be lucky enough for Dr. Shockley, conduction is the one-dimentional prolem ; this is one of my temporal conclusion. In fact, if you would read between the lines of Dr. Shockley's article, "The Path to the Conception of the Junction Transistor", IEEE Transactions on Electron Device, Vol. ED-23, No. 7, July 1976, p 597-620, [Ref. 44] and look into the copys of his Laboratory

Note Book therein, you would notice that he was thinking the electron motion in one-dimentional way of thinking.

I pointed out in the ◆◆COMMENTS in §2 of this article, that one of the choice that Solid State Physics to survive from the lethal blow of Topology is to stick to the "1-dimentional concept". Also, I pointed out that the only other alternative, in case you don't like the 1-dimentional world, is to get ahead as "The Local Physics".

Again, Dr. Shockley was ingenious and he had intuitively get to these points. He made the famous "Filamentary Semiconductor Measurements" for the Life Time measurement and Drift Mobility measurement for the injected carryer.

Incidentally, I recall the grievous days, when I was looking for the way to analyze the Anisotropic Mobility Data for the Photo-generated Carriers in CdS. All the peoples around me, theorists and experimentalists, was talking about the accepted idea of the Fermi-sphere shift due to the applied electric field.

When I shaped up by employing the "Lorentz Equation", instead of the Boltzman Equation, one of the Professor at the site became a green eyed monster. He said, "Your equation is just for the Single electron ! There's no concepts at all for the velocity distribution ! You have only ONE electron that rolling down through the crystal! While, that we call "One Electron Approximation" means we have N-Electrons'-Wave-Function using Single Electron Fnction !"

I knew that nasty business of the lengthy determinant formalism, with the most delicate shifting idea of the Fermi Surface. But, it did not give me any glimpus of getting out of the problem. Sure, one of T-JSSP gave me the Formula of Angular Dependence for Microscopic Scattering Angle. However, he kept silence when I asked him how to apply it to the macroscopic crystal anisotropy date. After all, the simplest method of Vector calculation employing Matrix algebra was succeeded in for me to get out the nasty problem [Ref. 45 : cf. especially Appendix for the present argument.]

Couples of months later, before the work appeared on Journal of Physical Society of Japan, I heard a promissing theoreical doctoral work was under way in another campus of Tokyo University, employing OUR data. This time, they said, things were going employing the decent theory of distribution and statistics into accout. However, they didn't give me any draft, nor reprint. I didn't hear amy rumor of spectacular success either. Luckily enough, I lost my interest very quickly when our article appeared on JPSJ.

I'm pleased very much to hear these days, that "Conduction is Transmission" [Ref. 46] Someone even says, "*So-called* conduction band", "*So-called* cyclotron experiment". [Ref. 14 ; p 157, 158]

Also on PHYSICS TODAY [Ref. 46], it reads as following ;

"The conventional point of view (held in the classical Drude-Sommerfeld or quantum mechanical Kubo theories) is that the electrical current density is determined by the local velocity distribution, which deviates from equilibrium in linear response to the local electric field. An alternative point of view had been put forward in 1957 by Rolf Landauer of IBM in Yorktown

Heights, New York ; he had proposed that "conduction is transmission" between reservoirs".

It was in 1957 ! Really ? If I knew the news, as of 1963, how much I would have been encouraged. Any one in the Institute, what they call the expecting Institute to produce the highest PEAK in Japan, did not let me know the news that should be appeared in IBM J. Res., 1957.

I remember clearly at the moment when I asked Professor Frederick C. Brown, whom I worked with in Physics Dept., Urbana, Illinois. It happened in MRL ;

"How do you think, if the equilibrium distribution of the photo-electrons, f(v), was not established ?", said I. Fred replied immediately, "All the story will breakdown !" I was moved, deeply, and thought, "Yes, he knows quite well where he is standing on !"

However, as of today, I can add some comments on our work ; "We were very lucky !" This is because, the method for the Transport Measurement employing photo-electrons, which was invented originally by Dr. Redfield and later modified by Dr. Brown, is One-Dimensional-Measurement. Dr. Redfield applied the magnetic field for the Hall-Angle Measurement, but he rotated the electric field so that the photo-current should resume the original direction. What's more, we did not need f(v) at all, in fact !

The photo-electrons have "very short" life-time : may be less than 1 m・sec. So, there should be a Source term and a Sink term, if someone would asked us to be a Perfect Formalist. At any rate, it is a laughable joke, that

a photo-electron must look around quickly, after his birth, which energy level he should occupy, so that he should not get a spank by Dr. Pauli ! Besides, he got to doublecheck all the crystal from $-\infty$ to $+\infty$, to the end of the Universe, before he would Die !

So, the small conclusion of this section is, OUR photo-electrons must had been a LOCAL Player. I won't say anything about other people's Transport measurements, nor another Subject in Solid State Physics, whatever it may be, since I have to save my Ammunition and Fuel for farther ahead targets.

Back to the Transistor Science, let me add just one very impressive comment given by Dr. Jack S. Kilby, Texas Instrument. He visited Japan, on March 1985. He attended the Science EXPO'85, which was held in Tsukuba-City. He came with his famous first test piece of IC, which he invented in TI (Texas Instrument). He answered at the Press Conference, held at Tokyo Hotel Ohkura. The digest of the talk was printed on the morning paper of The Mainichi (which means "Every Day"), on March 18, 1985. By answering the questions after the talk he replied ;

"As for the Research and Development of such an important thing as IC, it is absolutely hard to attribute its success to anyone. It is the results of everybody's contributions who joined the projects from all around." He was 61 years of age at 1985. And it was so nice to hear, that he spoke of Dr. Honey and Dr. Noyce in FC (Fair Child Semiconductor). Close to his end of talk, he added also, "I was too conservative for estimating the speed of expansion on IC."

Later in 1988, I received a letter from Dr. Ednor M. Rowe, SRC (SynchrotronRaditation Center) Wisconsin, US. He mentioned, "I never expected that this business would get so big——we were just trying to do something useful so that we could continue to work and eat!" [Ref. 47] Needless to say, he who knows the relations between the two fields of Science or Technology, would understand the meanings of the above two statements quite well.

§7 Dr. DIRAC got a Real Continume, but you don't!

In this section, I will discuss how different it is between "Real Continume" and "Almost Continuous".

★First of all, it is a very elemental knowledge on Differential Equation that the Eigenvalues will show up either DISCRETE or CONTINUOUS SPECTRA (and both of them, sometimes). This is solely due to Boundary Conditions ; the same equation gives different spectrum once the boundary condition were changed.

★Secondly, from the Physics point of view, the Eigenvalue Spectra of QM are categolized into as follows [Ref. 32] ;

momentum operator→continuous spectrum
angular momentum operator→discrete spectrum
Hamiltonian of hydrogen atom→both spectrum
atom and nucleus of atoms→both spectrum, (they say).

♣Here's where argument starts ;

1) You can change even the *discrete* spectrum *continuously,* only if you can change the boundary condition Continuously. Very simple. For example, shift the wall of the Microwave Cavity continuously, by turning the microscrew. If the Q-value of the cavity is high enough, you can take out different line spectrum of microwave out of the microwave generator. In this case, you have the standing wave solution between the walls of the cavity. Naturally, you'll get the higher harmonic mode waves, as the Fourier's Harmonic (integer) Theory shows.

In this case, the distance between the wall of the cavity (L) is Real Number (Aleph 1). If you would object that the microwave cavity analogy is too coarse, I am very glad to invite you to the Fabry-Perot interferometer. The scanning of etalon is achieved these days by computer controlled piezoelectric system, which let you allow to move by the atomic interval distance, as is evidenced by the scanning tunneling microscope (STM).

2) However, you CANNOT change the length of the crystal continuously. We are not talking of the cutting machine business. We are talking of the electrons' wave-function within the crystal. Here is the point of all sort of confusion inherited from the KNABEN PHYSIK (Kids' stuff Physics).

Seeing is believing : Why don't you look at Fig. 10, after M. C. Escher (1898-). Our world is exactly like this. There is no distinction at all, between the Boundary and the Bulk. (Let's refrain from talking about Dr. Bardeen's surface state). Within the microwave cavity or the Fabry-Perot, there is nothing but the empty space. (I found what a stupid-buy I did, when the Fabry-Perot etalon was delivered. I bought just an empty space

for $2,000 !) However, the crystal is not empty at all. You cannot escape from the Watching-All-Eye Fish, wherever you may go !

What's more, you cannot put your Boundary Surface somewhere between the lattice point. However much you may try to cleave alkali halides so that your Boundary Surface should appear in some place between the lattice point, you would just find yourself a loser, sitting in front of a heap of chanks out of kilo KCI. If you don't believe me, why don't you ask any one working with STM or MBE (molecular beam epitaxy)? Similar Kids' stuff

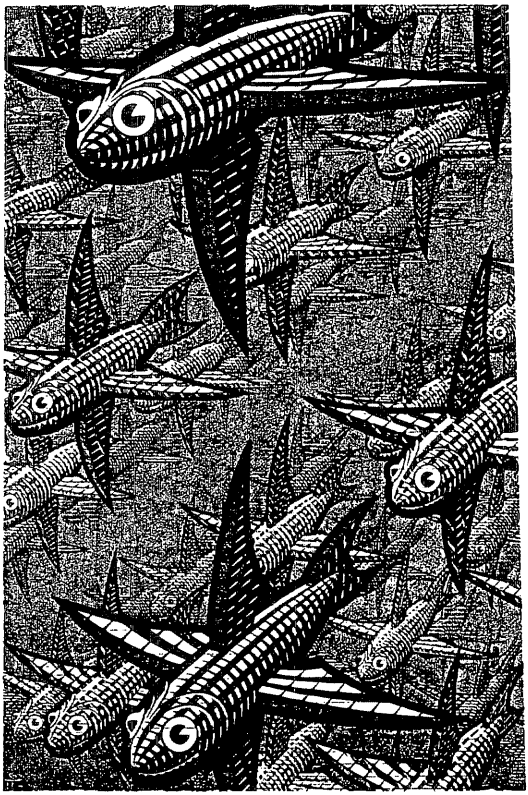

Fig. 10 The Escher's Fish. ("The Graphic Work of M. C. Escher", Meredith Press. New York. Fourth impression 1969 : Gifted from Prof. F. C. Brown)

is all around the DENKEN EXPERIMENT of KNABEN PHYSIK, such as X-ray microscope or double slit story. However, I have no time to share for them right now.

Dr., Bloch and Dr., Heisenberg, picked up the crystal length (L) for the fundamental harmonics, and put the highest harmonics (cut-off frequency) at the atomic interval (a ; L = Na). They must had looked at the success of X-rays people, and should had known well about the historically reviewed works introduced within Dr. Brillouin's book for phonons. 【Ref. 48】

By so doing, they have "got clamped" the electron wave-function at the lattice cites. As I showed you in "Bloch, Who?" (No. 1), actually they have separated the wave function further into x,y.z-axis. So they have got the "running peaks" due to the interference, which makes people feel fainting. (I don't know whether they realized this). They did not forget to put Atomic functions at the lattice point, which clamped the wave function at the mesh point. Everything went well, so they deemed, and the 3/4 century have passed.

3) I won't repeat the Storys to show you the DEFECTS. It must be drowsy to hear any more. Instead, I'd just point out how the above way of thinking contradicts with the modern Math.

I've already shown the conclusions of Topology in the previous work. I showed you in this work, the Measure of your Lebesgue integral is zero. And, the future task with C^0 functions. However, there are more to be mentioned :

♣Firstly, infinity (∞) is NOT a real number (R) any more.

We should have realized this at 1947, when Dr. Schwartz, Laurent (F : 1915-)(シュワルツ)discovered it. It is confusing, however, there was another Schwartz, Herman Amandus (G : 1843-1921) (シュヴァルツ), who is famous for Function Analysis with Complex Variables. People of modern Math write it straight down, as [Ref. 17] ;

$$\infty \notin R$$

which means Infinity is NOT A REAL-NUMBER. Infinity is an In, finity. We realized at last, , that we can never get to it (∞) by keep going. Since, there is no place at all on the Real Number Axis (if you prefer 1-dimensional world) that welcome you saying, "IT'S BEEN REAL !"

Über den Bergen,	Over the mountains,	山のあなたの
Weit zu wandern,	Far away to go,	空遠く
Sagen die Leute,	People says,	∞住むと
Wohnt das Glück.	There lives Happiness.	人の言う
Ach, und Ich ging	Ah, then I went	ああ我
Im schwarme der Andern,	With eager another,	人と求め行きて
Kam mit werweinten Augen	Came back ***sweet***	涙差しぐみ
Zurück,	Weeping eyesight,	帰りきぬ
Über den Bergen,	Over the mountains,	山のあなたに
Weit, weit drüben,	Far, far more to go,	なお遠く
Sagen die Leute,	People says,	∞住むと
Wohnt das Glück……	There lives Happiness……	人の言う

Kerl Busse　　Tr. HF　　『上田敏訳』による

♣Secondly, Distribution (超関数) is discovered.

By the discovery, character of δ-function has become crystal clear. [Ref. 32] To make long story short, there are two categorys on Distribution ;

1) Normal Distribution (正規超関数)
2) Singular Distribution (特異超関数)

1) Normal Distribution is defined as following,
when f is Measurable and $|f|$ is Integrable ;

$$F(\phi) = \int_{-\infty}^{+\infty} f(x)\,\phi(x)\,dx, \qquad \phi \in D.$$

2) Singular Distribution, for example $\delta(x)$ is defined as,

$$\delta(\phi) = \phi(0)\ !$$

Everything became crystal clear. I've never understood quite well (almost but not quite) about δ-function, by way of the lengtly idea employing the "limit". But now, look, it is standing at only $x = 0$, and δ-function is defined only by ϕ! Whatever the shape it maybe! May it be Thunderbolt or Black and White! It is much better than to struggle with the never ending ripples of $(\sin x)/x$, nor $\exp(i\omega x)$. It is far much better than to worry about the staggering steps! (Très claire!) Don't worry if δ has the "width" to integrate. Because the "Test Function ϕ" is standing upon the "carrier (or support)", i.e., on a Closed SET, where ϕ cannot be zero. [Ref. 17]

As the byproducts, let me add just two points ;

1) The Fourier Transform Equation is easily derived from the Distribution, and it is unnecessary to derive from the vague limit of Fourier Seriese.

2) Dr. Neumann disliked the Dr. Dirac's δ-function. He didn't use the δ, but went through the old-fashioned way. He just mentioned some comments for not employing δ ;

"We do not desire to follow any further here this train of thought which was shaped by Dirac and Jordan into a unified theory of the quantum processes. The "improper" function (such as δ (x), δ' (x)) play a decisive role in this development——they lie beyond the scope of mathematical methods generally used, and we desire to describe quantum mechanics with the help of these latter methods."

At any rate, if you would hear anyone to say, "Dr. Neumann mentioned about the usefulness of δ-function, on the way to verify the identical validity for both Dr. Heisenberg and Dr. Schrödinger's formalism", then you should understand that the man didn't see Dr. Neumann's book even up to p 27.

♣Thirdly, you will easily get outside of the Hilbert (L^2) Space !

May be Dr. Neumann could have realized this ; δ-function has got outside of his favorite space. Thanks to the distribution, we can calculate that the Norm of δ-function is [Ref. 32] ;

$$\int_{-\infty}^{+\infty} \delta^2 (x)\, dx = \delta (0) \rightarrow \infty, \text{ (Wow!)}$$

This is serious, since Dr. Neumann is said that he invented "The Separable Hilbert Space" in such a way ;

The BEIN (Basis) must be Complete Orthonormal with Finite dimension, or Countable-Infinite dimension, space. Another words, Any function f(t) within the Space L^2 can be expanded by the Fourier expansion such that,

$$f(t) = c_1 \phi_1 + c_2 \phi_2 + \cdots\cdots = \sum_{n=1} c_n \phi_2$$

which is the JSSP's favorite Priciples of Superposition (重ね合わせの原理). However, here f (t) must be Lebesgue square Integrable one, i.e.,

$$\int_{-\infty}^{+\infty} | f(t) |^2 dt < \infty.$$

Obviously, δ-function got out-side of the Dr. Neumann's Space, as shown above by $\delta(0) \rightarrow \infty$!

Not only that, any Eigen-function, in general, that belongs to the continuous spectrum is said to get out-side of the Space 〖Ref. 32〗. I guess you've found out where you are. The more you insist, the more the route got narrower. You are sleeping between the devil and the deep blue sea. What if you'd get out to the outside? There's waiting for you Non-Commutative, Un-bounded operator (discontinuous functions), and highly possibly Chaos and EVERYTHING !

♥Dr. Dirac's continume.

It is not hard to understand Dr. Dirac's work, which was published in 1928, employing the modern knowledge 〖Ref. 49〗. The non-Japanese readers who may want to be sure on the following argument are recommended to consult themselves to Dr. Dirac's original papers. 〖Ref. 50, 51〗

Almost in the same period, in 1929, Dr. Bloch published his work. While Dr. Lebesgue's new way of integral was almost completed at the moment. However, there is no trace in the Bloch's work about Lebesgue integral, as we have already seen.

Dr. Neumann published his work in 1932. 【Ref. 52】 He mentioned about Lebesgue integral, in the translated Princeton Book, at the foot note on p 24. 【Ref. 53】 However, it is not clear enough whether he applied the method 100% in the New Princeton book. The foot note did not appear in the Original German text. It is therefore, not clear whether Dr. Neumann did not hear the new integral, or he thought it is unnecessary to employ it. Since, all the function he used is Regular. Thanks to the progress of mathematics, we can make clearer arguments than those Professors.

Back to Dr. Dirac's work, in order to make a short-cut, let's remind the Relativistic Energy Equation is writen down as ;

$$E^2 = (mc^2)^2 + (cP)^2. \qquad \text{[eq. D-1]}$$

Then the corresponding "Wave Equation" due to the above Energy Equation is,

$$\{E^2 - (cP)^2 - (mc^2)^2\}\,\phi = 0. \qquad \text{[eq. D-2]}$$

Because, we take it for granted to replace E and P into the Operators,

$$E \rightarrow ih\,\partial/\partial t, \qquad P \rightarrow -ih\,\nabla,$$

which operate upon the Wave Function, ψ (r, t).

The [eq. D-1] is good enough for us to discuss the points of argument. However let's follow a bit to the Physicists' arguments. They say as following ;

Firstly, the Wave Equation must be order-1 for the Time Derivative, $\partial/\partial t$. This is because The Schrödinger Equation is standing that way. We will be completely at a loss, if we start with 2nd-order $(\partial/\partial t)^2$ equation. Since in this case, we don't know about what sort of the values or function is required to the $[\partial\psi/\partial t]$, for the Initial Condition.

Another Formalist climbs on the bandwagon. He declares, "*All The Information* must be contained within ψ (r, t). $\partial\psi/\partial t$ must not be given! It must be derived from ψ afterwards! This is the Essence of Wave Mechnics !"

So, now they restart from the point where the External electro-magnetic field is applied. The classical Wave Equation, which correspond to [eq. D-2] is,

$$\{(E + eA)^2 - c^2 (P + e/c \cdot A)^2 - (mc^2)^2\} \psi = 0, \quad \text{[eq. D-3]}$$

They *assume* that the Dirac's Equation which corresponds to [eq. D-3] must be ;

$$(E - c\alpha \cdot P - mc\beta)\psi = 0. \quad \text{[eq. D-4]}$$

After manuipulations of [eq. D-4], by substituting the operator form of E and P, and emplying the following commutation rule, with some reasons, such as ;

$$[\alpha, P] = 0, \quad [\beta, P] = 0, \quad \text{and } [\alpha, \beta] = 0,$$

finally obtains the following DIRAC'S EQUATION :

$$\{E + cA_0 - c\alpha\,(P + e/c \cdot A) - mc^2\beta\}\,\psi = 0, \quad \text{[eq. D-5]}$$

where Wave Function turn out to be 4-dimensional SPINOR.

If you apply the [eq. D-5] to the Hydrogen, you will get the Fine structure spectrum due to the relativistic effects.

Our purpose is not to discuss Dr. Dirac's Positron Theory, but to examine the discontinuity of the Energy curve vs. k (momentum or wave number), or vs. r (real cordinate).

As it may clear from [eq. D-1] or [eq. D-2], E is linearly dependent on P, and P has the continuous axis. This is for the free electron, or positron. You will see the situations are Essentially unchanged when we get to the relativistic Dr. Dirac's equation [eq. D-5].

When you get to Hydrogen, you will have the discrete energy levels. (plus Plank's constant uncertainty business, plus zero point vibration affairs). Whatever the matters they may be, you see, Dr. Dirac's Energy curve is Standing on the continuous real number axis (r). Dr. Lebesgue doesn't care

whether Dr. Dirac's calculation may be performed upon the continuous or discontinuous axes. However, Dirac's δ-function demands its Carrier (Suport) to be continuous closed set. And it is, with the best lucks, saticefying the demands.

Needless to say, for your case, things were not so lucky enough.

§8 Temporal Conclusions

So far, we found the following conclusions ;

1) From the Topology concered considerations (No. 1), The Solid State Physics that employs The Bloch Wave can be valid only in, "1-dimensional world". if you demand your platform from $-\infty$ to $+\infty$ on the axis.

2) You can play with the Game of Dynamics or what-so-ever you want, provided that you don't forget the following conditions ;

   [A] You have the Limited (bounded) Area of 2-, or 3-dimensional world. However, the Local area can be either small or large as you may want. But (!), you can never be able to extend it up to infinity (∞). Because, the infinity is not a number ($\infty \notin$ R) since 1947.

   [B] You have to prepare for the Fixed points may come out, once you start the Wave to circulate. Some fixed point may be associated with Chaos.

The following temporal conclusions should be added, as the results of this work (No. 2) :

3) When you come across the Summation ($\sum$) over k-axis, you have to be very careful. Since, if you would proceed along the line to Integration ($\int$), which the Traditional Solid State Physics had came up so far, then suddenly you would find your integration ends up with Zero. This is because you have not payed good attension for the Lebesgue Integral. I should just say here, "Something must be definitly wrong, at someplace, somehow !"

ACKNOWLEDGEMENTS :

The author's gratitude should go to his old friends, since Junior High School boyhood, by whom the author always resumed his firmness and tranquility :

Toshio Matsumoto, Professor of Information Design, Kyoto University of Art and Design, who pointed up the Spirit of This Era was the Post Modernism : Which let people to re-search and pick up the old jewels from the muddys, where those Modernists threw them into the discard. I'm afraid the author is getting on. However, I still want to roll up my sleeves and to keep up with those vanguard.

Yasukuni Tsuchihashi, the retiree of Tokyo Nishi Highschool Staff, whose soft endurance was always stick around the author, ever since we were struggling for existence upon the ash land of Tokyo. Especially, his sense of music and art always remind the author "What is creativity ?"

Takaji Kuroda, Corporate Chief Engineer, NEC Corporation, and The Member of The International Astronautical Congress. His cultivating ability to every direction of technical problem, always encouraged and amused the author.

The author also owes innumerable amount of colleague-ship and goodwill abroad :

Frederick C. Brown, Professor of Physics, University of Washington, WA, USA ;
Robert Bachrach, Director, Dr., Applied Materials, California, USA ;
Christian Gähwiller, Dr., Hoffmann-La Roche, Switzerland :

All did not react with the flat negative rejection to the former curiously strange article, "Bloch who ?". Their kind neutrality gave the author the strength to stick to the gun.

Other old colleagues, Charles Pruett and John Budden, retirees of SRC Wisconsin, gave the author invaluable quantity and quality of old friendship : john's clear cut Veteran troops' judgement, and Charlie's gentle warm friendship ; the author doesn't know the words to express his thanks. Actually, this woek. No. 2, was started in his quiet lovely guest room during the Rededication of Ednor M. Rowe Lab., which Charlie kindly offered the author to stay from May 2, to 6. 1997.

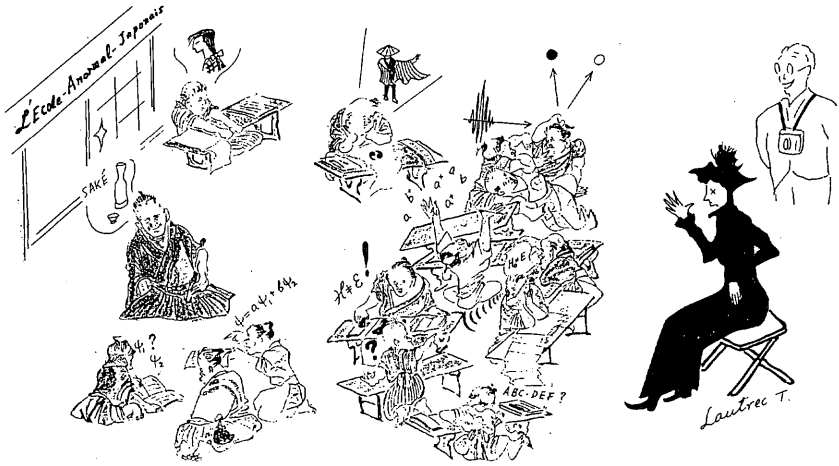

Fig. 11 A picture of a private class (Tera-Koya) in Edo-Period; by Kazan WATANABE (1793-1841): 【渡辺華山・寺子屋の図】

The most popular period of Tera-Koya teaching system was from 1804 to 1844, as shown by this picture. There were more than 900 class rooms in Edo(Tokyo) city, whose population was about one million. Teaching job is a sort of "private enterprise" (tax-free). Teachers were, what you call, "half-time staff".

Their main jobs were; the lower class Samurai (warrior), monk, jobless warrior, and for some case, professional lady teacher. No license was necessary. Average educational fee ranged from 1/4 Ryo (=250 Mon) to 300 Mon. Incidentally, a bowl of nuddle costed about 18 Mon. So it is about 25-luncheon cost-equivalent; economically very poor. Of course they cannot afford the living cost, by the teaching income only. Someone invented "semester system", or even quarterly, so that they can multiple the income.

However, fee was not fixed socially. It was flexibly determined, depending upon the economical conditions of the parents. For some case it was achieved completely free. Of course the parents paied later, either by cash or by some stuff for the daily use, when it became available to compensate.

The size of the class was from 13 to 600. No age limit to enroll. However normal-

ly it started from age of 7. All the classmates and the teacher were the neighbors and they knew each other quite quite well. No calicurum, no exams, no qualifications. Because it was invented to teach and to learn; not to select pupils. What did they learn? Well, why don't you ask Mr. Watanabe. I don't remember that far!

You may notice the teacher in left end is holding a strange stick within his left arm. That is The Short-Sword, which indicates his status (warrior : Samurai). You may get some knowledge on a cut-away cross section of the Japanese Culture, when you would read the famous "Chrysanthemum and Sword".

All was gone with the wind, when The New Goverment took over Edo (Tokyo).

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(Dr. Robert T. Beyer wrote the "Translator's Preface", at the date of "December,

1949". Dr. Neumann was found very sick in the summer of 1955. He was kept on a wheel chair since January 1956, and past away February 8, 1957. Dr. Neumann should had enough time to notice that his old work is going to be revived. My never ending question is, as I wrote in No. 1, "Why Dr. Neumann didn't make any comment on the New Book, as the "Author's Preface" ? "May be, I'm afraid, Every American knows Everything. However, Any Japanese doesn't know Anything !)

Jack and Jill
Went up the hill,
To fetch a pail of water ;
Jack fell down,
And broke his crown,
And Jill came tumbling after.

Fig. 12 The man of La Mancha, and his troop. (after Gustave)