NOTE

Etude No.1: "Who killed the Foucault-Pendulum ?"

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CONTENT : Abstract

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Dr. Berry's article is examined. In the article, he mentioned about the Foucalt-Pendulum. However, his way of description is incorrect and coarse. Therefore, the Foucault-Pendulum Problem is pursued until its ultimate solution within the framework of Classical Mechanics. This is because the problem is expected to have relations with the Berry-Phase Problem in Quantum Mechanics.

It turned out, however, that the problem is much harder and more fundamental than the author's anticipation. For another effort, Dr. Stone's related article is examined by calling forth the knowledge of Topology. It turned out, again, that the work is very heavy burden and heat of the day.

After all, the precise discussions on the Berry-Phase Problem is carried on to the following work, Etude No.2.

§ 1 Introduction

Dr. Berry's article appeared on PHYSICS TODAY of December issue in 1990, from page 34 through 40. 【Ref. 1】 The article reads a lot of sparkling spots even as of today. However, it appears that the article containes improper examples. This is not Dr. Berry's fault. It is due to us Physicists' and/or Geophysicists' fumble. (We didn't dig up The Classical Mechanics until the final ultimate stage, did we ?) Hope this little etude would stimulate someone to find a junction for getting on to Quantum DYNAMICS from the Classical Mechanics.

In § 2, The Foucault-Pendulum Problem is handled via the up-to-date knowledge of Classical Mechanics. This is because, the problem is surely unrealistically handled in Dr. Berry's article.

In § 3, a preliminary but a critical argument is given to the Berry-Phase problem, from the Topology point of view. More detailed arguments will be appeared in this circular; Chuo-Gakuin-University (CGU) Report.

In § 4, discussions for the above two sections are presented, once again. Temporal conclusion is given in § 5.

§ 2 The Foucalt-Pendulum

Dr.Grosmann, University of Louis Pastoul, Strasbourg France, recommended to read Umberto Ecco's book of "The Foucault Pendulum". 【Ref. 2】 Author has no opportunity to have read the book yet. However, it is the author's guess that the book may not have the arguments which we are going to roll out.

【Fig. 1】 shows Dr.Berry's Foucalt-Pendulum which appeared on PHYSICS TODAY. 【Ref. 1】 The pendulum is approximated by a TRIANGLE. This is, however, "The Over Simplified Approximation". It may sound like perfect when you consider the ratio of the globe radius to the amplitude of the swing; say 10m to 6300km. Here is, however, the improper start line for the approximation.

How can a pendulum do swing within a horizontal plane ? The Foucalt-Pendulum is not a conical pendulum, which swing along a horizontal circle.

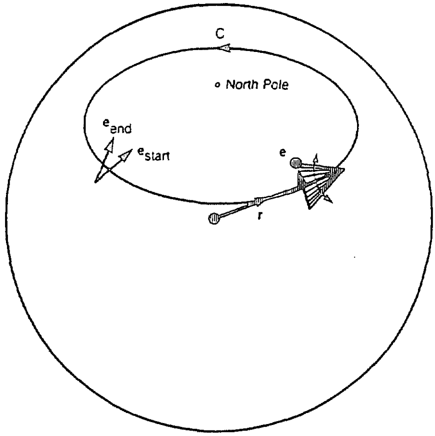


Fig. 1 "Dr.Berry's Foucalt-Pendulum", which appeared on PHYSICS TODAY, Dec, 1990. Notice that he regards it as a Triangular Pendulum. This is obviously improper assumption.

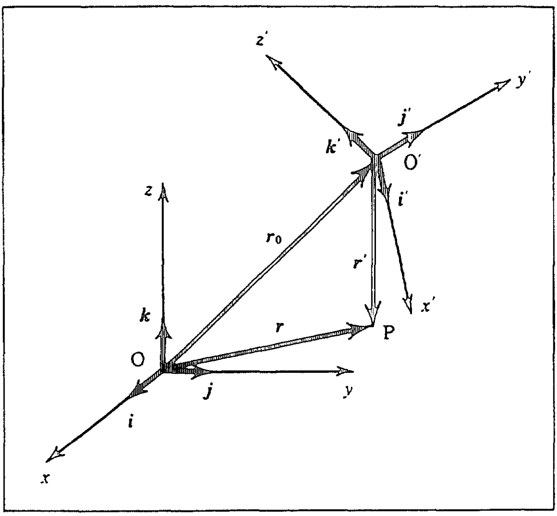


Fig. 2 "The Relative Coordinate system in Classical Mechanics". The importance of the system was repeatedly mentioned in the latest work; "Who afraids of Born-Wolf ?" Incidentaly, the author of the text, Dr. Hori, is an Astro-Physicist.

The Trajectory of the motion of Foucalt-Pendulum cannot be such one.

Here is a precious thing, by the way, that The Modernism discarded into the garbage tank; author said this comment a thousand times. (千度〈センド〉言うた) Hope, readers realize how important it is to recognize the Relative-Coordinate System. Why don't we start from the 【Fig. 2】, which is shown as Fig.41 in the last work; "Who afraids of Born-Wolf ?"

As shown in 【Fig. 3】, the Foucault-Pendulum is a spherical pendulum. By the word "Sphereical", author means the Trajectory is spacially limited by a sphere. 【Ref. 3】 Actually, this is a Tolus Problem as shown in 【Fig. 4】, since the globe is rotating. This is a very tough problem. Question is by which system, o-or-o', the problem should be handled. Of course, they can be transfered by not-so-simple matrix to each other. The Foucalt-Pendulum does swing in the Non-Inertial system (非慣性系), which is a really tough problem.

The fundamental equation for a particle which moves on the rotating sphere is as following 【Ref.4 】;

$$F = m\,[\ddot{r}_0 + (D^2 r'/Dt^2) + 2\omega \times (Dr'/Dt) + \dot{\omega} \times r' + \omega \times (\omega \times r')] \quad \cdots\cdots\cdots\cdots (1)$$

where, F is the force which acts on the pendulum,

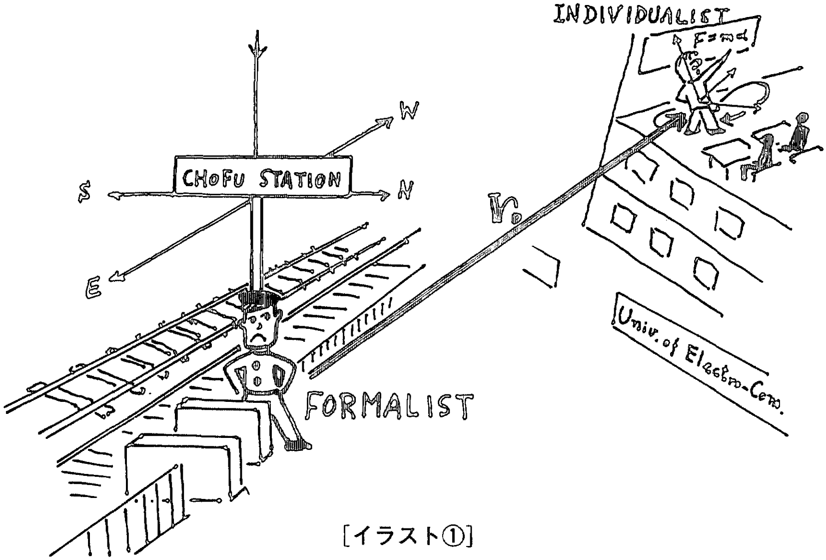
m is the mass of the pendulum,

r_0 is the radius vector of o-system,

r' is the vector of o'-system,

ω is the rotation vector of the globe.

The derivative D/Dt is defined by



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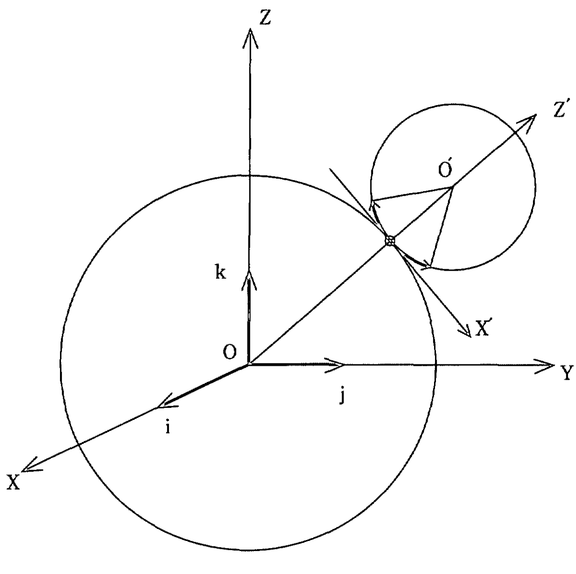


Fig. 3 The Foucalt-Pendulum is a Spherical-Pendulum. It cannot be replaced by the Conical-Pendulum. Since the approximation of that way will lose everything.

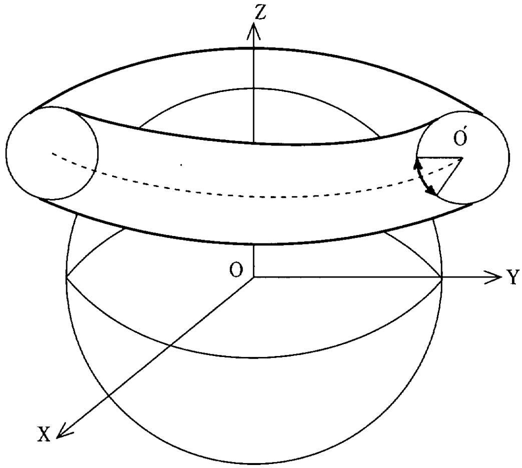


Fig. 4 The trajectory of the Foucalt-Pendulum is limited by the Tolus-Surface. Any Simple-Pendulum is, actually, Tolus-Pendulum. There is not a "Simple Pendulum" on the earth.

$$dr'/dt = Dr'/Dt + \omega \, x \, r' \quad \cdots\cdots (2)$$

which is called convective derivative,sometimes. For our case,

$$\omega = \omega k, \quad \cdots\cdots (3)$$

$$\dot{\omega} = 0. \quad \cdots\cdots (4)$$

Then we have for the fundamental equation,

$$F = m \, [\ddot{r}_0 + (D^2 r'/Dt^2) + 2\omega k \times (Dr'/Dt) + \omega k \times (k \times r')] \quad \cdots\cdots (5)$$

The term $2\omega k \times Dr'/Dt$ $\cdots\cdots$(6)

is called the Coriolis' Force,

and the term $\omega\, \mathrm{k} \times (\mathrm{k} \times \mathrm{r}')$ ··(7)
is the centrefugal force due to rotation of the globe.

The equation (5) may sound like a simple linear differential equation for r'. But the complicacy comes to the surface, when one realize that there is mixing up with two systems; o-and-o'.

As you see well, r' is a vector that belongs to o'-coordinate, but r and k belongs to o-coordinate. Therefore we got to transfer the vector k into o'-coordinate system. The transfer matrix for the job is as following;

$$\begin{pmatrix} \mathrm{i} \\ \mathrm{j} \\ \mathrm{k} \end{pmatrix} = \begin{pmatrix} \sin\varphi.\ \cos\theta, & -\sin\theta, & -\cos\varphi.\ \cos\theta \\ \sin\varphi.\ \sin\theta, & \cos\theta, & -\cos\varphi.\ \sin\theta \\ \cos\varphi \quad , & 0, & \sin\varphi \end{pmatrix} \begin{pmatrix} \mathrm{i}' \\ \mathrm{j}' \\ \mathrm{k}' \end{pmatrix} \cdots\cdots (8)$$

The final results are obtained by the straight forward calculation. The equation of motion is written down as following, in o'-coordinate, for x'-, y'-, and z'-axes;

$$\mathrm{m}\, \mathrm{x}' = \mathrm{F}'_x + 2\mathrm{m}\omega.\ \sin\phi.\ \dot{\mathrm{y}}' + \mathrm{m}\omega^2\, \mathrm{R}.\ \sin\phi.\ \cos\phi \quad \cdots\cdots\ (9)\ \ \text{(to the South)}$$

$$\mathrm{m}\, \mathrm{y}' = \mathrm{F}'_y - 2\mathrm{m}\omega.\ (\ \sin\phi.\ \dot{\mathrm{x}}' + \cos\phi.\ \mathrm{z}'\ \ ) \cdots\cdots\ (10)\ \ \text{(to the East)}$$

$$\mathrm{m}\, \mathrm{z}' = \mathrm{F}'_z + 2\mathrm{m}\omega.\ \cos\phi.\ \dot{\mathrm{y}}' + \mathrm{m}\omega^2\, \mathrm{R},\ \cos^2\phi \quad \cdots\cdots\ (11)\ \ \text{(upward)}$$

where we made the approximations, which can be well justified this time;

$$\omega^2\, \mathrm{x}'\ ,\omega^2\, \mathrm{y}'\ ,\omega^2\, \mathrm{z}' \ \ll \omega^2\, \mathrm{R}. \cdots\cdots (12)$$

As you see it well, equations (9), (10), and (11) are mutually connected. As the result, it is easy to foresee that it will end up with Non-linear equation. It is worth reminding the Chaos-equation, which couples altogether. It is obvious that this problem cannot be solved without computer. Also it is very instructive to remember that the Foucalt-Pendulum is the "Energy-Non- Conserving System". Author wants to mention more about this subject in § 4.

§ 3 The Berry Phase

Dr. Berry cited the works of Dr. Stone 【Ref. 5】 and Drs. Mead and Truhlar. 【Ref. 6】 It appears that these works have full atmosphere of "Mysticism". Maybe it is bacause the author is not a chemist. However, once the door was opened, the reality is not-so-complicated. Their mathematical tools are almost the same as The Color Center Physicist. More closely speaking, it is the same as the Ligand Field Theory, or LCAO (Linear Combination of Atomic Orbitals).

For example, they handle the collision (or chemical reaction ?) process of the "Three Atoms". More precisely speeking, they handle the collision between hydrogen atom and hydrogen molecule as following;

$$H + H_2 \ . \qquad \cdots\cdots (13)$$

They won't try to solve the 4-body problem. Their business starts as same as the Ligand Theory. They pick up the wave functions of atomic hydrogen; be it s-orbitals or p-orbitals. They believe in that they can construct the Total Wave Function. So, their wave function ends up with the product of "Nuclear part and Electronic part". Now they employ the "Born-Oppenheimer Approximation. This is to separate the electron-coodinate from that of the Nucleuses. This is called Adiabatic Approximation, also.

The reason why author said “Mysticism” is they appear to know how to twist or distort the three-body atomic system. Author really dont know how to distort three body system, since “Three points always determin a Plane”. But whatever heck it may be, they must be right.

However, when it comes to “Configuration coordinate-Q”, author cannot hide his suspicion. 〖Ref. 5〗 True, Q-coordinate was quite useful idea, and it was powerful for the F-center physics; especially for the pioneer work. It was not long before, however, people began to ask “What does your Configuration-Coordinate-Q really mean ?” This sense of Abstract-Expressionism is sticking around to this Q-coordinate, too.

The problem is, they plot the “different configurations on to a plane”. They call it as a “Circle on a Nuclear Configuration Space”. Here, they claim the analogy to the Foucault-Pendulum. Readers would realize, however, that this analogy is not proper at all. Since the motion in the “Q-configuration space” does not depend on time [t]. At any rate, the author's friendship came to the end, when there appeared “A phase-preserving surface”. It is shown on 〖Fig. 5〗

As you see it, this is just another example of “Continuous Projection”, which author argued in “Bloch who ?”. Hope, readers would remember that author denied the existence of 3-dimentional Bloch-function in Euclidean space.

Author won't raise any argument about the quality of the “surface” on 〖Fig. 5〗. However, whatever the surface maybe, it cannot escape from the criticism of Topology; as long as it concerns about “Space”. Before we argue Topology,

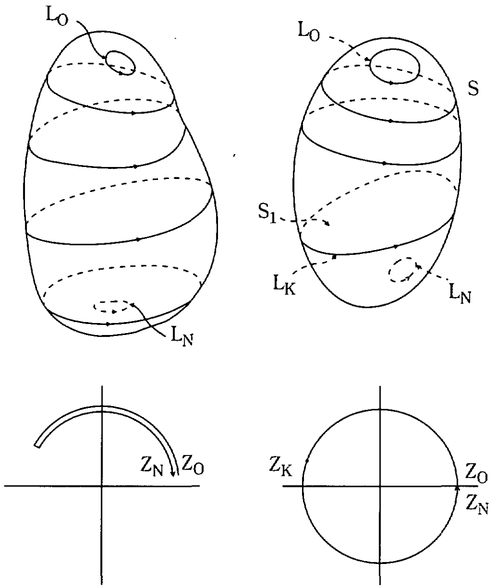


Fig. 5 Dupulication of Fig. 3, which appeared in Dr. Stone's article; Roy. Soc, London 1976 【Ref. 5】 Readers may feel a sort of difficulty to swallow the hardness, to connect "the Volume" and "the Z-plane". However, it is easily understood if everything was viewed from the view point of Topology.

here is one thing that author have to offer for the our convenience; it is the list of the interpretation of the terminology, from that of chemist to physicist and mathematician's. They are,

Transport → Projection

Parallel transport → Continuous (neighbor to neighbor) projection

Umbilic → Source and/or sink

Nuclear → Atom (or proton) in (hydrogen) molecule

arbitrarily small → infinitesimally small

(they don't know the infinity does not belong to real number)

As you see 〖Fig. 5〗, this is again the "continuous projection". By the word "continuous", it means that the "neighboring point is projected to another neighboring point on the new map". For example, suppose you have moved from Madison, Wisconsin to Urbana, Illinois. And if you found your next neighbor moved to your New Next Door, then you should call this "Continuous Projection". Because this is deeply concerened with "Continuity of a Function".

Not only this, as author descrived in "No.1, Bloch who ?", there must be FIXED POINT on the surface, as shown in 〖Fig. 6〗. Therefore, what they call "Umbilic" is nothing but "The sink which was generated by drilling a hole upon the FIXED POINTS".

Now you see, once you've got a hole, then the body in the Q-coordinate can be changed into a Tolus, and finally into a disc which has some thickness. What's more, they neglected the size of molecules. They cannot make the Umbilic "arbitrarily small", because molecules have the finite Size. They can-

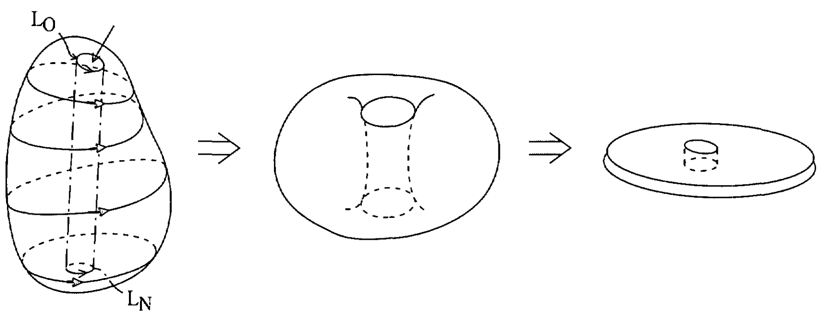


Fig. 6 Topologicaly equivalent sequence from "The Volume to The Disc". When one may drill a hole through the Fixed Point, then one can get to the other side of the disc. No big discussion nor lengthy excuse are needed.

not beat Topology. If they claim it is the size of the order of Angstrome, we can magnify it up to the size of the universe. Problem is to study the quality of space; not on the size.

Therefor, the argument they rolled out about the nature of the curvature of the space around the umbilic is just a Pleasant Nonsense. Author refrain from argument about the mysterious Cone-and-Potential topics at the point. Loughable thing is, however, some T-JSSP climbed up the bandwagon and emploied the Cone-business with relation to the Aharonov-Bohm Effect arguments on suspicious electron microscope experiments.

§ 4 Discussion

We restrict our discussion on the Foucalt-Pendulum. The arguments on Berry-Phase will be carried on to coming work, "Etude No.2."

【Fig. 7】 shows the bird's-eye-view of the Foucalt-Pendulum. The pendulum gets the maximum speed at point [P] . Thereby the ball gets the maximum Coriolis' force which is perpendicular to the arc [A-B] . The Coriolis' force pushes the pendulum ball toward the point [R] .

However, because of the sphereical curvature, the ball runs up the slope about up to the point [Q] . Mathematically speaking, the arc [A-Q] is a 3-dimentionally winding curve, and its Bi-normal is Non-zero. 【Ref. 7】 Besides, the Coriolis' force vector [C] changes its direction and magnitude from point to point successively. You might imagine, this is a very complicated problem, and we cannot handle it without computer.

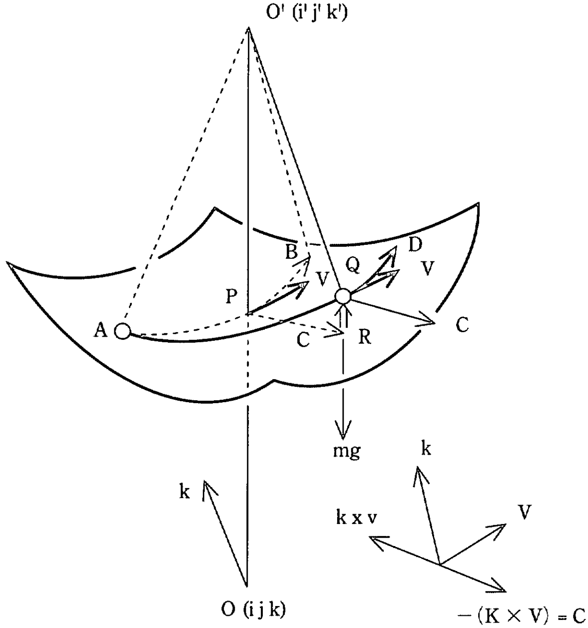


Fig. 7 An exagerated bird's-eye-view of the Foucalt-Pendulum. The trajectory of the pendulum is 3-dimentional curve. There is a Non-zero bi-normal. What's more, this is Non-adavatic problem.

However, as we will see later, the Coriolis' force is about 0.01 gal at the point [P]. So, let's assume the Coriolis' force vector "keeps its original direction and magnitude [C] at the point [Q], as shown in 〚Fig. 7〛.

It would be easily imagined, as shown in 〚Fig. 8〛, the Coriolis' force sets forward the rotational motion of the Foucalt-Pendulum around the vertical axis [k']. This effect is popular for us Japanese, since there is no year that is free from spiral windy Typhoon.

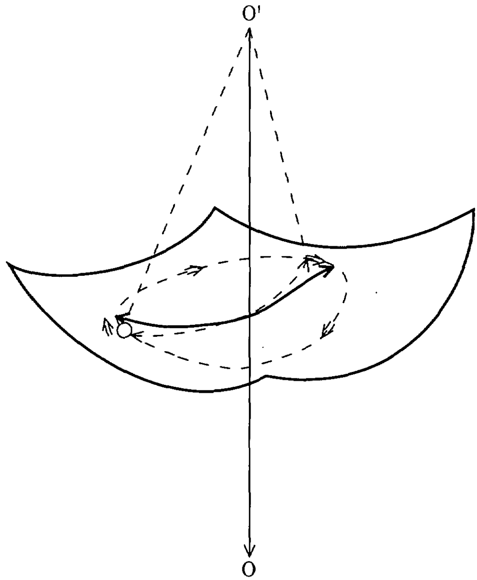


Fig. 8 The Coriolis' force which get the Foucalt-Pendulum to rotate. It is Non-linear, 3-dimentional, and Non-Energy-Conserving system. What's more, the Coriolis' force is CUMMULATIVE. The accumulated action become dramatical amount after 24 hours (86,400 sec) of elapse.

Question is "How strong the Coriolis' force can be ?" We got to estimate the strength of the force. 【Fig. 9】 shows the instantaneous balance of forces. The estimation goes as follows;

The formula that we need is [$2\omega \times (Dr'/Dt)$].

$\omega = 2\pi / 24 \times 60 \times 60$ (sec) $= 7.27 \times 10^{-5}$/ sec

$Dr'/Dt = 1$ m / sec (assumption)

The answer is 1.45×10^{-2} gal.

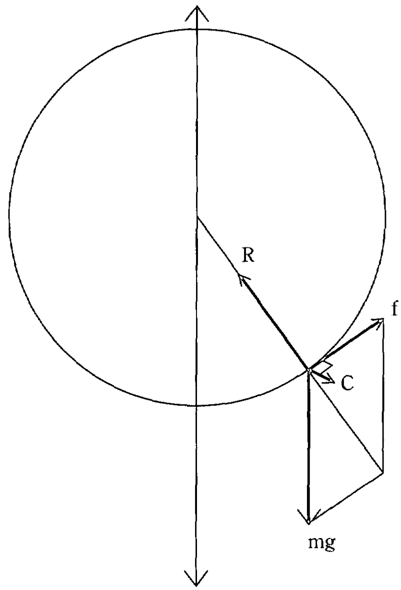


Fig. 9 An exaggerated figure for "Balance of forces". The actual Coriolis' Force is order of 1/100 gal a swing. However it is accumulative, as stated in the text. As the natural gravity is about 980 gal, so the ratio of the two forces makes 1×10^{-5}. However, as the Chaos people know it well, Coriolis' force acts all in-phase. That makes the big results.

Readers may regard that 1×10^{-2} gal is neglibly samll when it is compared to 980 gal; the gravity of the globe. However, the point is, this effect is CUMULATIVE. For example 24 hour makes 86,400 sec, and this factor must be multiplied to 1.45×10^{-2} gal.

Answer is 125.53 gal a day. Now you would agree this is a great number.

This is something like Chaos. The Coriolis' Force acts on the Pendulum always In-Phase to the swing; which generates the big rotational angle after a

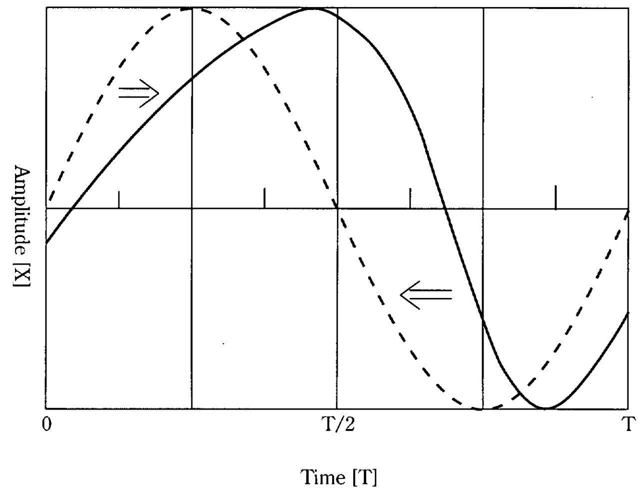
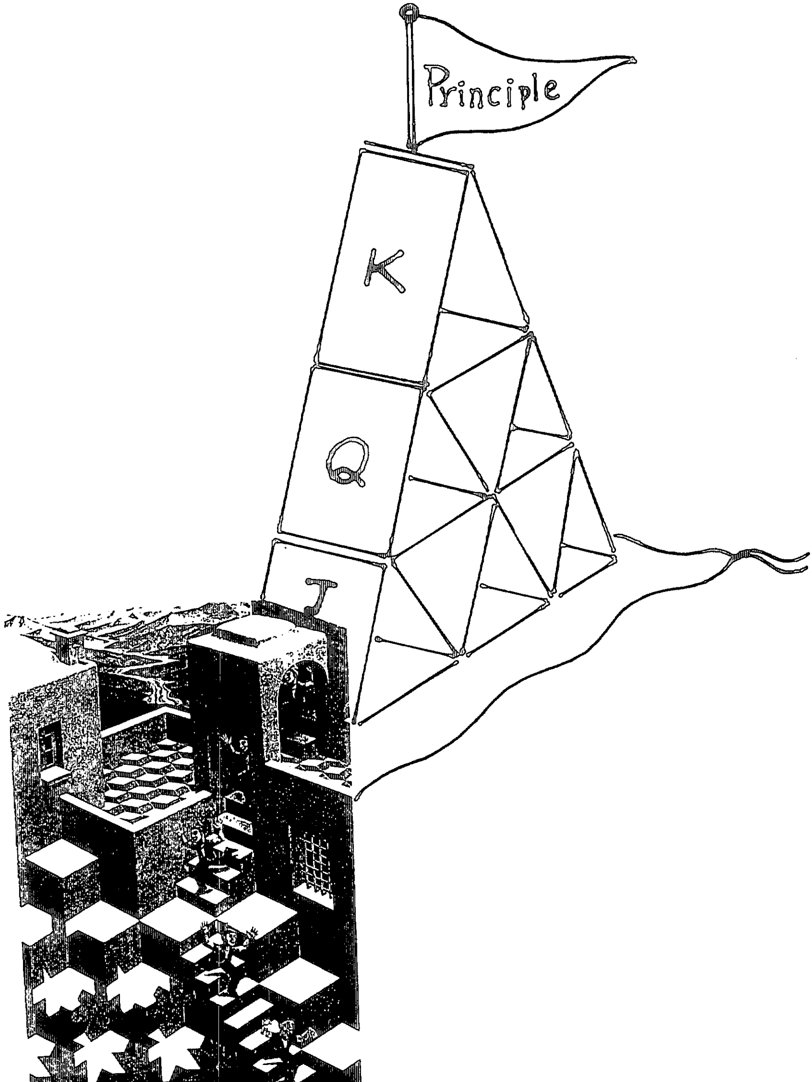


Fig. 10 The speculated trajectory on the [x - t] chart. Non-linearity is exaggeratedly shown. The wave form is squeezed, little by little, until it may become flat, and it will be rotated by π-radian.

day.

The situations are shown in 【Fig. 10】 by exaggerated speculation. As is shown, the first phase when the Foucalt-Pendulum swings to the North, the Coliollis’ force acts to deviate the swing to the East. Then in the back-swing phase, the Colliolis’ force again acts let the swing to delay in the x-t space. After all, the curve that started by the Simple-Sinusoidal trajectory can be distorted as shown. This is the Non-linear effect.

So, the deviation-angle from the normal trajectory for each swing can be observed if the observation accuracy were escalated up to 1×10^{-5} degree. It is due to this Non-linearity that makes people to notice the big-deviation after a day.



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The last question is where this energy comes from, so that the pendulum to rotate? Of cource from the globe's momentum or the energy of rotation. Therefore, the Foucalt-Pendulum is Non-Adiavatic System. This is the problem which was discarded into the garbage tank by "Theoretical Physicists". However, Geo-Physicists are those who never forget that the globe is rotating. It is very plausible that after, say 100 years later, all the Junior-Hi boys and girls would burst into laughter, when they heared about 20th century's Analytical Classical Mechanics (解析力学).

§ 5 Temporal Conclusion

The Foucalt-Pendulum problem is pursued until its ultimate analytical solution. It is found the system is Non-linear and Non-energy-conserving. The Berry-phase problem is slightly touched, and the topological error is pointed out with regard to their space-concept.

[**REFERENCES**]

(1) M. Berry; PHYSICS TODAY, December 1990, p.3−40.

(2) M. Grosmann; Private communication.

(3) M. Shinada, Y. Tanabe; "Analytical Mechanics", Sohkaboh, 1990. (in Japanese) 品田, 田辺 ; 『解析力学』 裳華房, 1990.

(4) G. Hori; "The First Lecture on Classical Mechanics", Maruzen, 1990. (in Japanese) 堀 ; 『はじめての力学講義』 丸善, 1990.

(5) A. J. Stone; Proceeding of Royal Society, London, A. vol. 351, p.141−150, 1976.

(6) C. A. Mead, D. G. Truhlar; J. Chem. Phys. vol. 70, 1, March, p.2284−2296, 1979.

(7) K. Yano; "Differential Geometry". (in Japanese) 矢野 ; 『微分幾何学』 数学全書, 朝倉書店.