NOTE

Etude No.2: On The Logic of Bertlmann's Socks and Bell Inequality

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Abstract

The Bell inequality is examined by using the set theory. It is shown that the derivation of the Bell inequality becomes very easy to understand. It is also shown that there are two types of logic which changes the connectivity number of the space by applying the Bell inequality.

For the quantum mechanical theoretical side, it is suggested to use the helium atom model for the Einstein-Podolsky-Rosen problem (EPR), instead of the conventional hydrogen-molecule model. It is stressed for this case that the spin-and-space coordinate is unseparable.

§ 1 Introduction

It is amazing to notice that more than half a century has past since Einstein-Podolsky-Rosen (EPR) published their article on Quantum Mechanics in 1935 [1]. What's more, it is interesting to find that the arguments are getting more and more exciting both experimentally and theoretically.

In the article of ref. [1], EPR presented the statement about the "Separability of the States". They refrained from mentioning explicitly, that "The Space can separate the States (or the Reality)". They did not mention straightforwardly "when the two phenomena were occurred with a good distance apart, then it is well guaranteed that they are independent even for the quantum phenomena, just like for the classical case".

EPR did not mention explicitly, either, that there is something which cannot stand with the special relativity, within the existing framework of quantum mechanics. EPR stated this idea so sophisticatedly that peoples are still very scarcely realize the EPR's thoughts.

It is Bohm who first succeeded in to write down the EPR problem by using mathematical equations, in 1951 [2]. He employed the spin function of the two particles, and simulated the EPR problem by a Thought-Experiment of the Stern Gerlach magnets. Then thereafter, people called the EPR problem as the EPR-Bohm's paradox [EPRB]. By the way, EPR never called the problem as the paradox.

Later in 1957, Bohm and Aharonov issued an article [3], and stated more clearly what is the problem of the EPR paradox. In the article of ref. [3], they pointed out two things:

【1】 The Theoretical Side.

Bohm and Aharonov summarized the Bohr and the Copenhagener's idea on this subject. They, the majority and the authodox people, BELIEVE that the quantum mechanical object and the classical system is one thing. They believe it is impossible to put any CUT at any specific point with any specific reasons between the quantum mechanical objects and the classical measuring instruments. It is a matter of AMPLIFICATION: the accuracy of the measurement for the quantum phenomena is not great enough. This is not only because of the technical problem. More than that: This is because there is the essential Uncertainty with the quantum phenomena itself. Therefore, the only thing we can expect to observe is to get informations by the statistical means. This does not imply to employ the classical statistics, however. They employ the PROBABILITY of the quantum mechanical system itself. They don't need the Local Causality, and in this sense, they don't need The Relativity, either. As a matter of fact, they don't see any PARADOX on the EPR problem. Someone was even brave enough to declare that he solved ALL The Paradoxes of the quantum mechanics.

To the contrary, Bohm and Aharonov, the minority and the heterodox, BELIEVE that there must be a new conception, which should be able to shed a light on the EPR paradox. They wanted to go with Local Causality and the Special Relativity. They suggested the two possibilities;

(A) The concept of the "quantum potential".

This is a new potential, which connect a distant particles, where there is NO Classical Potentials. The concept is not fine enough so far, and it is still unknown whether it satisfies the relativity or not.

(B) The concept of the "sub-quantum mechanical theory".

This is a new theory which is in the lower subquantum-mechanical level. The necessity of such a theory is obvious since the EPR problem indicates that there is a correlation between the quantum properties of distant things. It becomes more necessary when one thinks about the "many-body problems", as Einstein is called that he pointed it out already.

【2】 The Experimental Side.

Instead of the Thought Experiment, using the Stern Gerlach magnets, they suggested to employ the "electron-positron annihilation phenomena", so that they can use the polarization measurements of the lights. Very fortunately, the mathematical formulations between the Thought spin Experiment and the light polarization experiment is rather similar. The idea to examine the EPR problem by experiments were carried forward. More over, in Appendix of ref. [3], they showed the EPR problem is quite a common phenomena in collision problems.

Later in 1964, Bell came in and offered the famous Bell inequality [4]. He succeeded in to formulate the EPR (classical) conditions into a mathematical formula. It turned out the result is different from that of the quantum mechanical result. Actually, the first article in 1964 is full of the "brain twisting" statements. However, the result was very encouraging, which showed the possibility to perform a critical experiments to examine the EPR statements.

In 1981, Bell restated the verification by more sophisticated way, and the way of his argument became clear [5]. In ref. [5], Bell started his argument by applying the "Wigner-d' Espagnat inequality" [6, 7], which is rather hard to swallow by a quick reading. It says as following;

"The number of young women is less than or equal to the number of women smokers plus the number of young non-smokers".

The author found it is very easy to understand this complicated logic when the elemental "SET THEORY" is employed. The author found also, there are two kinds of logic within the Bell's article, which obviously he did not realize. So, it is the purpose of this small article to show peoples how it is promising to use The Set Theory to EPRB problem. It is also the purpose of this work to point out that we should be careful to categorize the EPRB problem into two parts; which is well known in the classical mechanics, however. (To begin with, the auther recommends for readers to consult the author's former work; "L'addition s'il vous plaît ! (No.3) Who affraids of Born-Wolf ?" [8]. Since there is a small summary of the introduction of the set theory.)

In 1982, there came Aspect-Dalibard-Roger's (ADR) experiment [9]. ADR was the first who tried the light polarization experiment by using the fast optical shutters. ADR was great in the sense that they switched from the imposible Thought-Experiment of the Stern-Gerlach magnet to the controllable optical polarization measurement. To the author's opinion, however, ADR experiment raised the questionable ambiguity, which is related with the special relativity. The author refrains this subject here, and it will be mentioned in a separate paper.

In § 2, the way to set up the problem is shown by employing the elemental set-theory. In § 3, the readers will be guided to the use of the easy applications of the set theory for the "Logics". In § 4, the above section is connected to the existing Bell inequality equations. The relation between the existing Bell inequality and the present result is summarised in the Table 1. In § 5, it is shown as the conclusion that the Bell inequality must be reconsidered more carefully.

§ 2 Set Up of The Problem

Bel tells us in his article of "Bertlmann's Socks and the Nature of Reality" [5], that "Dr. Bertlmann likes to wear two socks of different colours. Which colour he will have on a given foot on a given day is quite unpredictable" (Fig. 1). Let's concentrate our effort on "The Logic" which Bell is talking about. There are three steps before we come to the inequality.

Bell starts from the "Wigner-d∏Espagnat inequality" [5], i. e., ;

"The number of young women is less than or equal to the number of women smokers plus the number of young non-smokers".

This is a very complicated logic. However, readers may easily understand

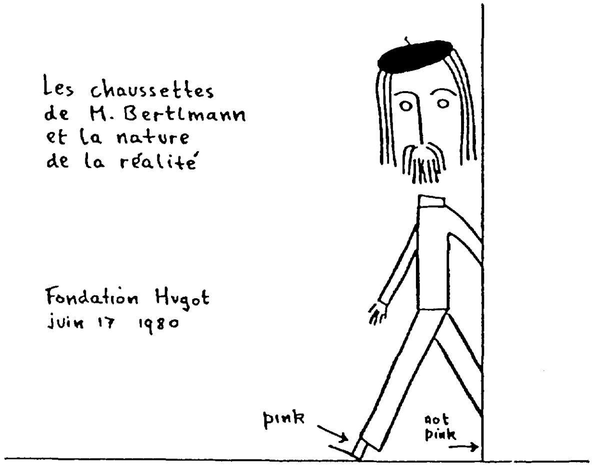


Fig. 1 Dr. Bertlmann and his socks.
[Dr. Bertlmann likes to wear two socks of different colors. Which color he will have on a given foot on a given day is quite unpredictable.]

when he consult the Fig. 2; this is the simplest application of the Set Theory. Bell's logic continues as following;

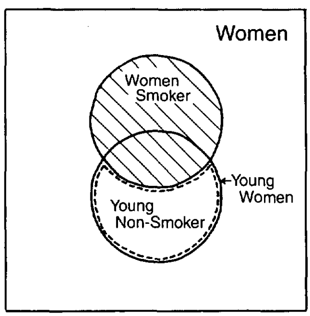


Fig. 2 Wigner-d'Espagnat inequality.
[The number of young women is less than or equal to the number of women smokers plus the number of young non-smokers.]

Logic No.1,

(the number that could pass at 0° and not at 45°)
plus
(the number that could pass at 45° and not at 90°) (5)
is not less than
(the number that could pass at 0° and not at 90°)

Logic No.2,

(the number of pairs in which one could pass at 0°
and the other not at 45°)
plus
(the number of pairs in which one could pass at 45° (6)
and the other not at 90°)
is not less than
(the number of pairs in which one could pass at 0°
and the other not at 90°)

Logic No.3

(the probability of one sock passing at 0°
and the other not at 45°)
plus
(the probability of one sock passing at 45° (7)
and the other not at 90°)
is not less than
(the probability of one sock passing at 0°

and the other not at 90°)

Logic No.4

(the probability of one particle passing at 0°
and the other at 45°)
plus
(the probability of one particle passing at 45° (8)
and the other at 90°)
is not less than
(the probability of one particle passing at 0°
and the other at 90°)

Logic No.5

(the probability of being able to pass at 0°
and not able at 45°)
plus
(the probability of being able to pass at 45° (9)
and not able at 90°)
is not less than
(the probability of being able to pass at 0°
and not able at 90°)

After stepping through these logical statements from the Logic No.1 to the Logic No.5, Bell presents a very complicated statement:

"And this is indeed trivial. For a particle able to pass at 0° and not at 90° [and so contributing to the third probability in (9)] is either able to pass at 45° (and so contributes to the first probability).

However, trivial as it is, the inequality is not represented by quantum mechanical probability". (cf. page CS-52 in ref. [5])

Here is the purpose of this work, i.e., to show people how these ENTANGLED statement become clear and understandable when the Set Theory is used.

The author does not care for the following distinctions; either it is on "the number (Logic No.1)", or the "number of pairs (Logic No.2)", or "the probability of one sock (Logic No.3)", or "the probability of one particle (Logic No.4)", or "the probability of being able to (Logic No.5)". The author does not care either the "degree (0, 45, 90)" means the angle of the Stern-Gerlach magnet or the "optical polarizer". The only thing that the author is interested in is "The type of the Logic of the Set Theory".

Let's start to set up The Set. We have the choice to get The Element of the set; it can be number, position, velocity, or function vector, i.e., the state vector. First of all, we have to define the Whole Set, which is represented by the square [X], as shown in Fig. 3. It should cover the whole area, where the Mechanics (be it Classical or Quantum) should take place within. (This is already stated in the Fig. 31-a, in ref. [8].)

Just for the sake of convenience of the arguments, let's take "the probability" for the elements of the set. As for the observable, let's take "the angle (θ)" of the instruments. Let's assume also that the domain of the arguments for "the element A" is represented by a rectangular box, as is shown in the Fig. 4. We

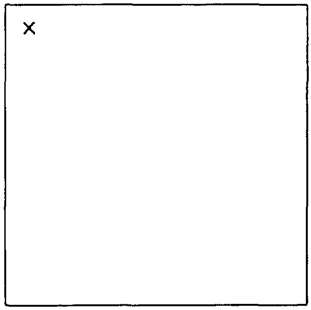


Fig. 3 The domain of SET (X)
[The domain contains all the element which belongs to the set X.]

can amuse by ourselves by changing the shape of the domain from rectangular to another shape. This will be presented in a different paper.

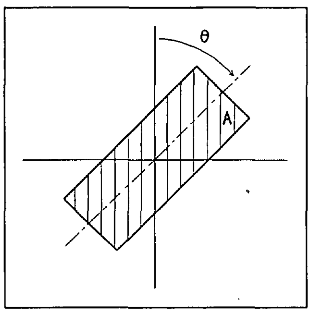


Fig. 4 The SET (A) that belongs to (X)
[SET (A) is represented by a rectangle, with a length (a) and the tilted angle θ to the axis.]

§ 3 Plotting The Logic

Let's plot "The Logic No.1 (eq. 5)" on the map. (We start by eq. 5, so that our equation number should meet with those in ref. [5].) The logic says:

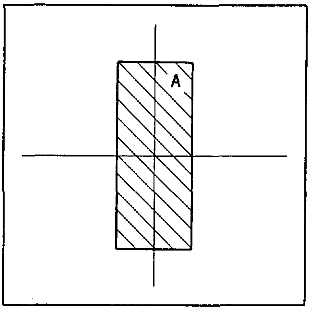
(The number that could pass at 0° and not at 45°)
plus
(The number that could pass at 45° and not at 90°) (5)
is not less than
(The number that could pas at 0° and not at 90°)

First of all, let's write this logic in a short form as following;

$$(0\text{-OK}, 45\text{-NOT}) + (45\text{-OK}, 90\text{-NOT}) \geqq (0\text{-OK}, 90\text{-NOT}) \cdots\cdots\cdots\cdots (10)$$

The author asks readers' greatest attention on eq. (10), since this is the most important corner where we have to turn.

To begin with, it is very important to know what "The Logic says" for a Set (A) and for a Set (NOT A). As shown in the preliminary article (ref. [8]), Set (A) is represented by Fig.5 (a), and Set (NOT A) is represented by Fig. 5 (b). This is very evident and "trivial". D'Espagnat says NO is orthogonal to YES [10]. However, NOT is the "compliment of the set" YES（補集合）. (cf. Appendix)



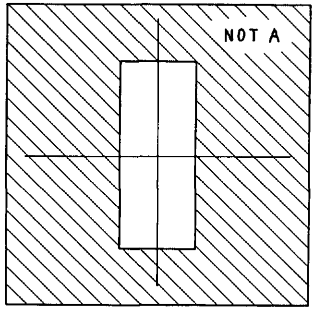
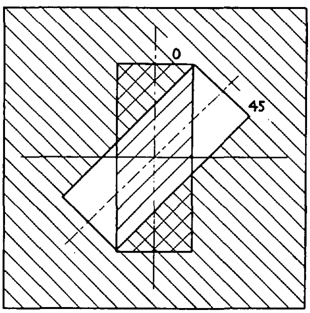


Fig. 5 (a) The domain of the SET (A)

Fig. 5 (b) The domain of the SET (NOT A)
[This is "the compliment of the set (A)" (補集合)]

Let's plot the Logic No.1 (eq. 5), which is represented by the short form as eq. (10), on the Fig.6 (a) and Fig.6 (b) ;

(0-OK, 45-NOT) → (0-OK) AND (45-NOT)



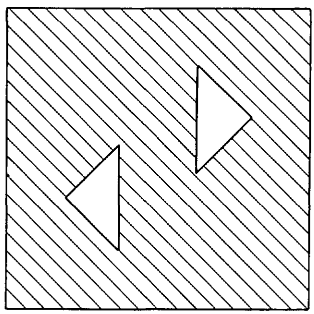


Fig. 6 (a) The domain of the SET (0-OK, 45-NOT)
[The overlaps of the domains are illustrated]

Fig. 6 (b) The domain of the SET (0-OK) AND SET (45-NOT)
[The same as Fig. 6 (a) ; however, the edge of each domain is erased]

The readers may feel something uneasy to see the Fig. 6 (b); we left with the two pieces of triangular openings like windows. Therefore, the first important thing about the statement (0-OK, 45-NOT) is that we have to realize it is not so evident nor trivial as peoples are talking about. To say "NO" is simple, but it is not that quite simple to say "NOT". In Fig. 7, the readers will see what the author is talking about. To say "No thank you" is just to erase the statement "Thank you". But to say "Not thank you" does arise lots of the troubles; readers may recognize this by the Fig. 7. (also cf. Appendix)

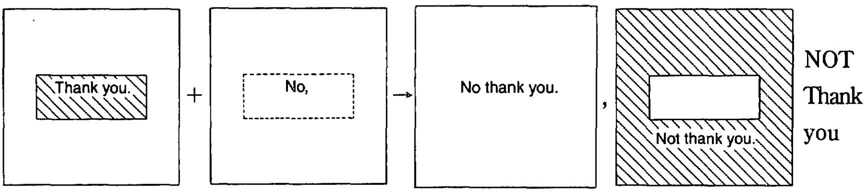


Fig. 7 The illustration to show the difference between the SET (NO thank you) and the SET (NOT thank you).

The second thing that the author is very anxious to call for the readers' attention is that we should notice the Two Open Space on Fig. 6 (b). This means that we have to work within the space where the connectivity is 3, instead of 1, i.e., the simply connected space. This is the harsh results of "The Observation". Therefore "The Observation Problem" is strongly connected with the change of the connectivity of space and "The Physics in Space". It is not that simple problem at all that the old theorists were thinking about.

The next step of Logic No.1 (eq. 5) is (the number that could pass at 45° and not at 90°), which is written in the short form as,

(45-OK, 90-NOT) → (45-OK) AND (90-NOT)

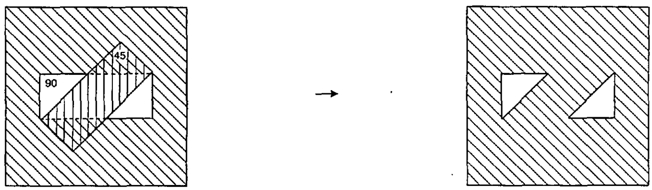


Fig. 8 (a) The domain of the SET (45-OK, 90-NOT)

Fig. 8 (b) The domain of the SET (45-OK) AND SET (90-NOT) [The composing process is the same as we have done for Fig.6 (b)]

Finally, the Logic No.1 demands to sum up (plus) Fig. 6 (b) and Fig. 8 (b), which goes as following;

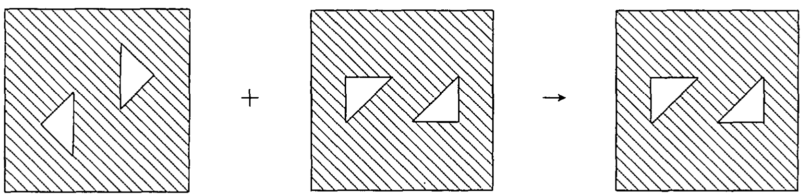


Fig. 6 (b) → plus → Fig. 8 (b) → Fig. 8 (c) [= Fig. 8 (b)]

The last statement of the Logic No.1 says "which is not less than (the number that could pass at 0° and not at 90 °)". This is shown in Fig. 9.

(0-OK, 90-NOT)

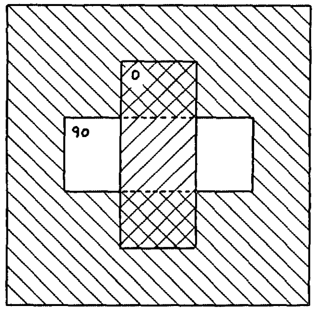
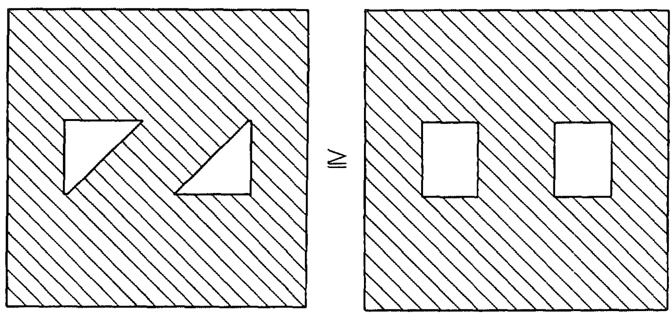


Fig. 9 The domain of the SET (0-OK, 90-NOT)

It is obvious from Fig. 9 that, Fig. 8 (c) ≧ Fig. 9.



The Bell's statement for the Logic No.1 says, after all, that Fig. 8 (c) is "not less than" Fig. 9, which is quite in accord with our results. Therefore, from the logical point of view, all the logic, No.1, No.2, and No.3, is the same.

More over, after some entangled arguments, Bell shows us the Logic No.4 (eq. 8) is shown as following;

(0-OK, 45-OK) + (45-OK, 90-OK) ≧ (0-OK, 90-OK). ……………………eq. (11)

Equation (11) can be picturized as shown in Fig. 10 (a, b, c). It must be obvious to any reader that, Fig. 10 (a) + Fig. 10 (b) ≧ Fig. 10 (c).

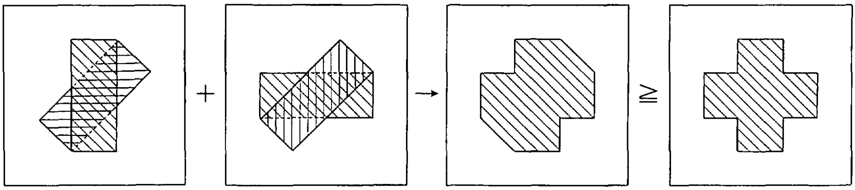


Fig. 10 (a)
The domain of the SET (0-OK,45-OK)

Fig. 10 (b)
The domain of the SET (45-OK,90-OK)

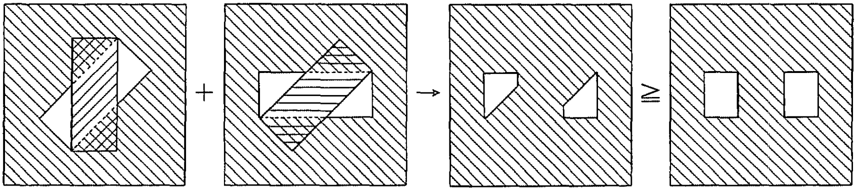
Fig. 10 (c)
The domain of the SET (0-OK,90-OK)

This means, of course, "greater or equal", and it is the same to "NOT Less Than". Therefore, again, the Bell's statement for the Logic No.4 is in accordance with our results. It should be stressed here, however, that the connectivity of space is just 1 for this case, which is called "singly connected".

The Logic No.5, which is the last one, is the same as No.1, No.2, and No.3. The statement says, "the sum of the two PROBABILITY is Not Less Than" the resulted probability. Therefore we can write it down as following, since "Not Less Than" is the same as "greater or equal".

(0-OK, 45-NOT) + (45-OK, 90-NOT) ≧ (0-OK, 90-NOT).

The readers can see easily on Fig. 11 (a, b, c), that the conclusion of the statement above is clearly "greater or equal", which is again "Not Less Than".



(0-OK, 45-NOT) (45-OK, 90-NOT) (0-OK, 90-NOT)

Fig. 11 (a) + **Fig. 11 (b)** → ≥ **Fig. 11 (c)**

The domain of the SET (0-OK, 45-NOT) The domain of the SET (45-OK, 90-NOT) The domain of the SET (0-OK, 90-NOT)

After all, the Bell's logic is all the same; "Not Less Than". However, the "CONNECTIVITY of the Space of The Set" is different between No.1, No.2, No.3, No.5, (which has connectivity 3), and No.4 (which has connectivity 1, i.e., singly connected space).

§ 4 Discussions for The Bell Inequality

Bell followed the "Wigner-d'Espagnat's Inequality" and found the famous Bell inequality [4, 5, 6, 7, 10]. Bell got the Logic No.1, and this was the very important logic. The structure of the Logic No.1 is the same as No.2, No.3 and No.5. For example, the Logic No.5 starts with the statement as following;

"the probability of being able to pass at 0° (OK) and not able to at 45° (NOT)".

Let's call this logic as (OK, NOT)-Logic, since it has the structure as shown by (OK) and (NOT) in the above statement. On the other hand, let's call Logic No.4 as (OK, OK)-Logic, because it starts as following;

"the probability of one particle passing at 0° (OK) and the other at 45° (OK)".

Table 1 summarizes the situations which we are facing with.
EPR-Bell-Bohm

I	Type of Logic	(OK, NOT)-Logic	(OK, OK)-Logic	Comments
II	Logic Number and Equation	No.1 (eq. 5) No.2 (eq. 6) No.3 (eq. 7) No.5 (eq. 9)	No.4 (eq. 8)	
III	Bell's Results	not less than [≧]	not less than [≧]	Bell inequality
IV	Results by Set Theory	greater than or equal [≧] ↓ not less than	greater than or equal [≧] ↓ not less than	
V	Results by QM Theory	less than or equal [≦] ↓ not greater than	less than or equal [≦] ↓ not greater than	
VI	Results by QM Experiment	?	?	cf. ref. 10
VII	Connectivity Number	3	1	for the first time

In Table 1, type of logic is shown in the first row, I. There are only two logics, i.e., (OK, NOT)-Logic and (OK, OK)-Logic. In the second row, II, the five logics (from No.1 through No.5) are sorted into two groups depending upon the type of the logic. As shown, there are four logics for the type (OK, NOT)-Logic. On the other hand, there is only one example for (OK, OK)-Logic, i.e., Logic No.4. This distinction is very important as we will see soon later.

In the third row, III, the conclusion of the Bell's statement is expressed "verbally". Bell's conclusion is the same for the two types of the logics, i.e., "not less than". "Not less than" is mathematically equal to the statement of "greater than or equal [≧]". We will use the mathematical signature [≧], when it is necessary, so that we should not be confused by using "the verbal expressions" only.

In the fourth row, IV, the present results are shown. The present results, which are derived by use of the Set Theory, are in accordance with the Bell's verbal result.

In the fifth row, V, the results of the quantum mechanical calculations are added. To the author's knowledge, the "theoretical results", which are derived by use of the quantum mechanics, are in contradictory to the Bell's prediction [5]: Bell said "not Less than", i.e., (greater than or equal [$\geqq$]), but the quantum mechanical results were "not greater than" (less than or equal [$\leqq$]).
This is the point where Bell thought that there maybe something weird about the quantum mechanics itself. Because, Bell inequality is derived from the sound logic of the classical mechanics.

However, as shown in the sixth row, VI, peoples of these days are trying to do critical experiments to examine the Bell's statements (cf. the examples in ref. [10]). They claim that their experiments were performed under the strict EPRB's locality conditions. Majority of the experiments agree with the quantum mechanical predictions. They call this as the "violation of Bell inequality".

Standing upon the experimental results, they claim that the Bell inequality does not work in the micro-physics. Not only that, they think that Einstein's demand for the Locality and Relativity maybe wrong. This is because the Bell inequality is derived by strictly satisfying the conditions for the classical mechanics, which Einstein was demanding. However, nobody trys to invent the reality that can transfer informations faster than the light velocity. This is obviously their dilemma.

To the author's opinion, however, we should be more careful enough before we get to any temporal conclusion. There are 5 reasons for this argument:

(1) Any modern experiment, which employs the polarized light pulses, appears to ignore the special relativity (Maybe they have no credit on relativity). The author will make critical comments in a separate paper on the polarized light experiments, which appeared after Aspect-Dalibard-Roger (ADR) [9].

(2) We should be careful enough for the connectivity number of the space. In the seventh row, VII, the connectivity mumber is indicated. (The author believes this is for the first time.) Obviously, Bell did not realize this point.

(3) As for the space which has the "connectivity 1", there is no need to argue about EPRB problem. Because in this case, the space can be shrinked continuously down to the atomic size. The necessity for the locality and relativity is well demonstrated already when the problem of "hyper-fine structure of hydrogen atom spectrum" was analyzed by classically and quantum mechanically [11, 12]. There is no conflict between the two results. The quantum mechanical result gives good explanations for the branching of the spectrum. On the other hand the classical concept of "the orbit and the mass of electron" represents the "local causality and the relativity".

Of course there is no way to put an instrument upon the atomic orbit of the hydrogen atom, nor there is a way to split the orbit. It is no need to argue about the EPRB-problem for this case. Einstein did not ask such a thing.

(4) When it comes to the problem within the space of connectivity 3, the physics becomes to be a matter of "boundary problem". Sometimes it becomes as the matter of "CONSTRAINTS", which is strongly limited by boundaries [13]. For example, the double slit problem, or the favorite interfer-

ence problem, is the harsh "Boundary Problem"; this is not the classical "Boundary Value Problem". The author pointed out in a previous work [8], that the cherished double-slit problem is a boundary problem, or the problem within the space of connectivity 3. This is a hard problem which is strongly connected with the scattering. It is like playing baseball game within a dome which has a lots of pillars.

To the author's opinion, EPRB-problem is the problem of which the space has connectivity more than 3. Einstein requested to separate the two states in space, which gives us the higher connectivity more than 1, unavoidably. This in turn, makes the problem as a part of "boundary problem", (not the boundary value problem). Another words, even in the Thought Experiment of the magnets, the particles must saticefy the "boundary condition" to "in-n-out" through the space between the pole pieces. There was no considerations about this situations when Bell showed us his numerical calculations by using his wave functions [5].

(5) Finally, it must be stressed here that "the theoretical quantum mechanical calculation" has a fatal error; it may be better not to say error, but it is a "hallucination" (錯覚). This is a matter of "mathematical physics".

The reason for this comment is as following;

It is Bohm who tried to set the EPR problem into mathematical physics. He picked up "Two 1-electron wave function with spin" [2]. Bohm showed by the popular spin combination method that we have to deal with the singlet state of "separated two particles' wave function". Bohm said in his book [2], "We have modified the experiment somewhat, but the form is conceptually equivalent by them, and considerably easier to treat mathematically."

In short, Bohm suggested to treat with "Hydrogen Molecule"; not with "Helium Atom". As is well known, the wave function of hydrogen molecule must be the solution of the "two-center integral problem". Conventionally, it is simplified by using the LCAO, and calculus of variation with 1-electron wave function. Bohm assumed further that an electron continue to move as a free electron after it is separated from the molecule. Another words, the electron wave function is assumed to be a simple product of a plane wave for the space coordinate, and the spin function. This is nothing but Bohm solved the problem before it must be solved. Therefore, it is natural that they say there is no change on the wave function, however much they may be separated [10]. They put the magnetic field, which is infinitely wide, upon these electrons. This is the essence of their thought-experiment.

However, how much the two electrons may be separated, this is a matter of "Three-Body Problem". At the beginning, we have to deal with two electrons within the same orbit. For this purpose, we need a nucleus to keep them within the atom. So, this is the typical three-body problem, of which Poincare stated already in 1892 that there is no general integrable solution. It took about another half a century before we understand what Poincare was saying; it was a part of "Chaos". Let's think about what will happen upon these two electrons when they are separated by elevating up their energy into the continume state.

There are two staggering stones for the old primitive thought-experiment:

(1) Overlapping Rydberg seriese.
(2) Non-separable wave function into the product for space and spin vector.

(1) Overlapping Rydberg seriese.

There are some exceptional cases of orientation which make the three-body problem to be analytically solvable. These are, the Lagrange's triangle and the linear orientation. The latter, i.e., "one dimentionally streched helium atom" may appear a funny toy for mathematical physicists. Actually, it is rather realistic and useful idea.

It is Bohr himself who tried first to solve the helium problem, but without success. He assumed "the independent particle model" for the electrons. This is a model which NEGLECTS the electron-electron CORELATION completely. In short, in this case, helium atom has two independent "principal quantum number (N and N')". The Rydberg seriese is represented by N and N', as following [14] ;

$$E\,(N, N') = -54.4\,\mathrm{eV}/N^2 - 13.6\,\mathrm{eV}/N'^2.$$

This is "even a good quantitative approximation to the helium atoms". It can explain the auto-ionization phenomena and the energy dependent ionization cross section to the continume states.

Further, by employing the following "one dimentional streched helium" one gets to Chaos in the continume states.

(2) Non-separable term for space and spin vector.

When the electron-electron interaction is taken into account, one gets the "one dimentional stretched helium atom". This is because of the repulsive interaction between electrons; the two electrons find their preferential places to the oposit side of the nucleus. With this model, we can write down the wave function for the "extremal Stark states, $| s(\rightarrow) \rangle$, $| s(\leftarrow) \rangle$" [14].

The "Stark states" wave function is explicitly expressed as following [14];

$$\langle r | n; n_1, n_2, m \rangle = (n^2 \pi^{1/2})^{-1} f(n_1 m: \varepsilon/n) \ f(n_2 m: \eta/n) \exp(im \phi)$$

$$f(n m: x) = (p! / (p + | m |)!)^{1/2} x^{|m|/2} L_p^{|m|}(x) \exp(-x/2)$$

$$| S_n(\rightarrow) \rangle = | n; n\text{-}1, 0, 0 \rangle, \quad | S_n(\leftarrow) \rangle = | n; 0, n-1, 0 \rangle$$

Therefore, due to the $x^{|m|/2}$ term, space and spin vectors are non-separable, where we have,

$$\varepsilon = r + z, \ \eta = r - z, \ \phi = \arctan(y/x).$$

One may say that, due to the quickly reducing exponential term, the non-separability of the wave function becomes almost negligible. And the effects of underlying continume become also negligible. However, we are talking about the PRINCIPLE; not the practical approximation.

§ 5 Conclusion

Finally, the author's temporal conclusion is as following;

(1) One has to quantitatively refine the Bell inequality, so that we can talk about the more fine argument about the inequality. For that purpose, we

have to clear up the logical path, so that everybody can understand the physical meaning and the process of derivation, more easily and clearly. For this purpose, the author believe that the application of Set-Theory is very effective.

(2) One has to recalculate the quantum mechanical probability for the Bertlmann's Socks with the current knowledges for the Three-Body problem. Simple and easy good old days with hydrogen molecule model is gone forever. We have computers and experimental data, which obviously Einstein and Bohr did not have.

(3) The use of the set theory is sure to make it easy to expand the dimensions of the set. Another words, there should be no difficulty for analysis of the "Entangled Trio Nonlocality Test" [15].

APPENDIX: NO, it is NOT.

In the article of "The Quantum Theory and Reality", Bernard d'Espagnat showed the way how to get to the Bell inequality [10]. The way he showed was, however, very complicated and headachy. It turned out not so awful, once it was understood. But still, it is not so clear, why one must follow the way he shows.

One of the reason why it appears so complicated is that he uses the logic [OR] and [AND] only. In the article, he says he uses the SET, but it appears that he is not so good at the set theory. The readers may found already that how easy it is to understand and to get to the Bell inequality, when the SET THEORY is fully used.

Meanwhile, author realized the origin of the darkness of d'Espagnat's article. D'Espagnat says "NO is ORTHOGONAL to YES". D'Espagnat uses only 0-degree and 90-degree for the magnet angle. So, he must had thought that the "compliment of a set（補集合）is orthogonal to a set". The readers may agree how it is easy to understand the logic when the SET [NOT] is properly employed. We will be sure to enjoy more, when we employ [NOR] and [NAND] logic.

In order to make it 100 % be sure to demonstrate the importance of the logic [NOT], author gives an very elemental example of application of SET [A] and SET [NOT]:

The English class of Japanese junior high school generally starts by reading such sentences;

Q. Is this a dog ?
A. No, it is NOT.
It is a wolf (rat, cat, etc.).

The author will show you how these sentences can be visualized by the use of the SET THEORY. Then the meaning becomes clear. Just follow the Fig. 12, 13, 14.

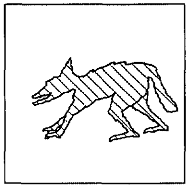


Fig. 12 (a) [Is this a dog ?]

★ For the Logic [NO].

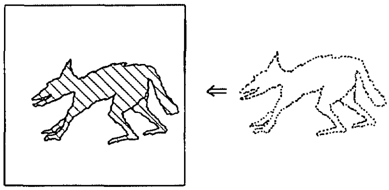


Fig. 12 (b) [No,] (click and draw)

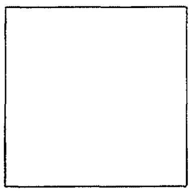


Fig. 12 (c) [it is erased/or disappeared)

★ For the Logic [NOT]

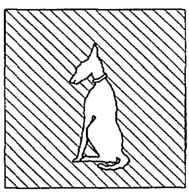


Fig. 12 (d) [(No,) it is NOT (a dog)]

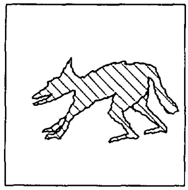


Fig. 12 (e) [It is a wolf (rat, cat etc.)]

★ Finally,

NO, it is NOT. It is a wolf, i. e.,

⟨ [NOT a dog] + [wolf] ⟩

⟨ Logic [NOT A] + Logic [A] ⟩

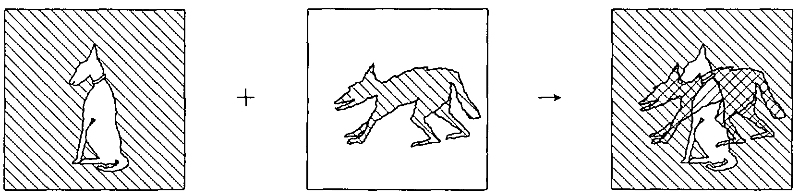


Fig. 12 (f) [No, it is NOT. It is a wolf.]

Therefore, ⟨ 45° [NOT] ⟩ + ⟨ 0° [OK] ⟩

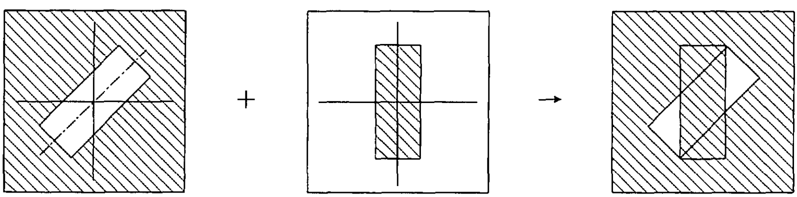


Fig. 13 [SET (45° [NOT]) + SET (0° [OK])]

The author is very pleased to show the readers, to whom it may not be agreed yet, a more elegant example. Please look at the following Fig. 14.

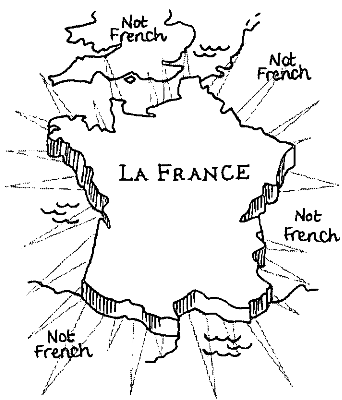


Fig. 14 SET (La France) + SET (NOT FRENCH)
[after "Xenophobe's guide to the French"]

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